Energy balance in self-powered MR damper-based vibration reduction system

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Abstract. The study is focused on the energy balance in self-powered vibration reduction system with a linear magnetorheological (MR) damper. The mathematical model of the vibration reduction system is outlined and numerical simulation data are provided. The model involves the mechanical and electrical sub-systems of the electromagnetic generator and an MR damper. For the purpose of the numerical simulations, the parameters of the model were identified through experiments. The objective of the simulations was to establish the energy balance of the system. Energy fluxes associated with energy inputs and outputs as well as a rate of change of inertial energy were determined.

Key words: MR damper, electromagnetic generator, vibration, energy balance.

1. Introduction

In the last decade, the field of energy harvesting from vibrations has increasingly become important [1–4]. There are many situations when mechanical energy present in the form of vibrations, converted into electrical energy, could be used to power devices. This work deals with the self-powered MR-damper-based vibration reduction system which employs the electromagnetic mechanism of electrical power generation from vibrations (Fig. 1). The system includes the electromagnetic generator and the MR damper. The system has adaptability by itself and does not require any controller, corresponding sensors or external power.
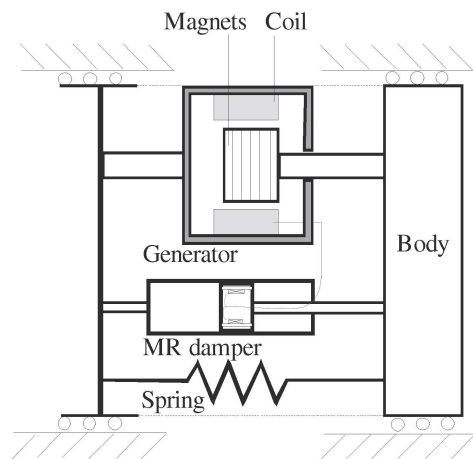

Fig. 1. Schematic diagram of self-powered MR damper-based vibration reduction system

The study discusses energy balance in the self-powered MR damper-based vibration reduction system. The system is composed of the vibration power generator incorporating high performance bulk magnets with special arrangement and foil-wound coil [5] and the commercial MR damper of RD-1005-3 series manufactured by the Lord Corporation [6].

2. Mathematical model

The vibration reduction system considered here is an electro-mechanical system. Schematic diagrams of the mechanical and electrical sub-systems are depicted in Figs. 2 and 3. The diagram of the mechanical sub-system shows its main components as well as the coordinate of the body 's position – x, applied kinematic excitation – z, the generator force – F_g, the MR damper force – F_d and the spring force – F_s. These forces act upon the body of the mass m. The model of the electrical sub-system comprises the connected coils of the generator and the MR damper. R_g and L_g denote the resistance and inductance of the generator coil whilst R_d and L_d are the resistance and inductance of the MR damper's control coil, e_g denotes induced voltage and i_g – current in the generator-damper circuit. The displacements and forces in Fig. 2 as well as voltage and current in Fig. 3 are depicted in such a way that it suggests the energy transfer from the mechanical to the electrical sub-system.
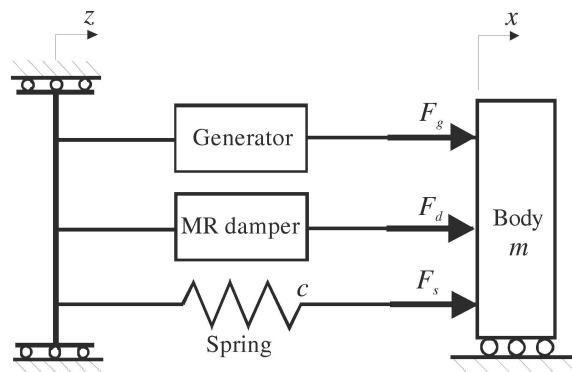

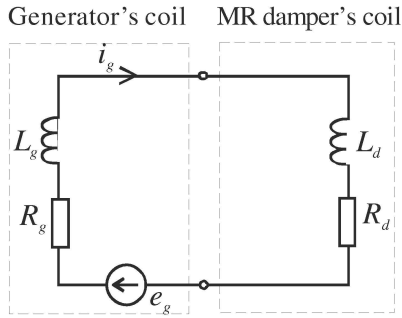Fig. 2. Schematic diagram of the mechanical sub-system

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Generator's coil   MR damper's coil

Fig. 3. Schematic diagram of the electrical sub-system

The motion of the mechanical sub-system is governed by the equation:

$$m\frac{d^2x}{dt^2} = c(z-x) + F_g + F_d. \tag{1}$$

Voltage induced due to the magnets' motion with respect to the generator's coil is given by the formula:

$$e_g = \kappa\left(\frac{dz}{dt} - \frac{dx}{dt}\right), \tag{2}$$

where κ is the generator's constant associated with the number of turns per the coil's unit length and with properties of the magnets.

Assuming that the conversion of mechanical into electrical energy involves no loss due to mechanical and magnetic phenomena and recalling Eq. (2), the force acting upon the body by the generator can be given by the formula:

$$F_g = \kappa\, i_g. \tag{3}$$

The force generated by the MR damper, whose direction is associated with the sign of relative velocity is given by the formula [7]–[9]:

$$F_d = (c_1|i_g|+c_2)\tanh\left[\beta\left(\left(\frac{dz}{dt} - \frac{dx}{dt}\right) + p_1(z-x)\right)\right] + \\ + (c_3|i_g| + c_4)\left(\left(\frac{dz}{dt} - \frac{dx}{dt}\right) + p_2(z-x)\right), \tag{4}$$

where c_1, c_2, c_3, c_4 are constants in the MR damper model, and β, p_1, p_2, are scaling parameters enabling transition, in pre-yield region, from negative to positive velocities. For $p_1 = 0$, $p_2 = 0$, $\beta \to \infty$, the model described by Eq. (4) is reduced to the Bingham model.

The electrical sub-system is governed by the equation:

$$e_g - (R_g + R_d)\, i_g - (L_g + L_d)\frac{di_g}{dt}. \tag{5}$$

Admitting the state variables: x – body coordinate, v – velocity of the body, i_g – current in the generator-MR damper circuit, and recalling the Eqs. (2)–(5), we get the state equations of the system:

$$\begin{cases} \dfrac{dx}{dt} = v \\[2mm] \dfrac{dv}{dt} = \dfrac{1}{m}\left\{\left[c(z-x) + \kappa\, i_g + \right.\right. \\[2mm] \quad + (c_1|i_g| + c_2)\tanh\left[\beta\left(\left(\dfrac{dz}{dt} - v\right) + p_1(z-x)\right)\right] \\[2mm] \quad \left.\left. + (c_3|i_g| + c_4)\left(\left(\dfrac{dz}{dt} - v\right) + p_2(z-x)\right)\right]\right\} \\[2mm] \dfrac{di_g}{dt} = \dfrac{1}{(L_g + L_d)}\left[\kappa\left(\dfrac{dz}{dt} - v\right) - (R_g + R_d)\, i_g\right] \end{cases} \tag{6}$$

3. Analysis of energy flow

An analysis of energy flow is one of the basic methods to research into the processes occurring in physical systems. Kinetic energy and potential elastic energy of the mechanical sub-system as well as the magnetic field energy of the coils in the generator and in the MR damper are functions of state and therefore they can be determined, by applying the solution to the state Eqs. (6) in a simple way. Dissipated energy is not a function of state and hence it has to be established by computing the relevant integrals. When analysing energy flows, it is more convenient to resort to dissipated power in order to eliminate additional calculations of integrals.

In the energy flow analysis, power is often considered as an energy flux. Figure 4 shows the energy fluxes, indicated by arrow symbols. Energy fluxes in the system associated with energy inputs and outputs are: P_{in} – input energy flux, P_{dh} – flux of energy transferred via the MR damper, P_{gh} – flux of heat energy produced in the generator.
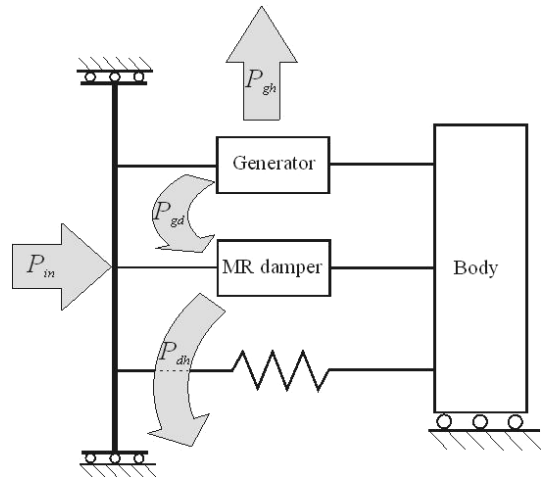

Fig. 4. Energy flow in the mechanical sub-system

Energy is supplied to the system from the shaker executing excitation z. During its motion, the shaker overcomes the resistance force of the mechanical system. The shaker's power P_{in} is given by the formula:

$$P_{in} = (F_g + F_s + F_d)\frac{dz}{dt}. \tag{7}$$

In the vibration reduction system considered in the study a part of mechanical energy is directly converted into heat energy, which is associated with rheological properties of MR fluid whereby the force generated by the MR damper at any instant has the direction opposite to that of relative velocity. Thus the flux of energy P_{md} taken from the mechanical sub-system (body and shaker) is given as:

$$P_{md} = F_d \left(\frac{dz}{dt} - v \right). \tag{8}$$

When balancing the energy fluxes associated with the generator and MR damper's behaviour, care must be taken to consider the energy fluxes in their electrical sub-systems as well. The energy flow in the circuit containing the generator and the MR damper electrical sub-systems is shown in Fig. 5.
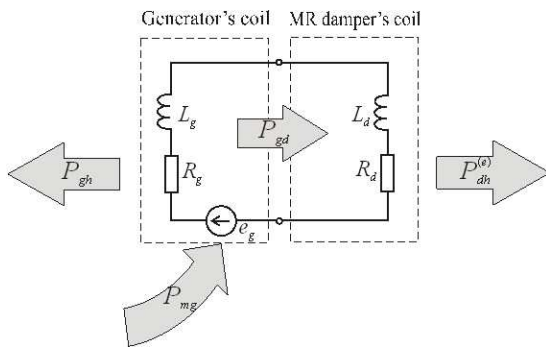

Fig. 5. Energy flow in the electrical sub-system

During the current flow, electrical energy is converted into Joule-Lenz heat. The relevant flux of energy in the generator is designated as P_{gh} and in the MR damper by $P_{dh}^{(e)}$. The energy flux $P_{dh}^{(e)}$ adds to the flux P_{md}, giving rise to the energy flux P_{dh} (Fig. 4).

As it was assumed the generator converts the mechanical energy into electrical energy without losses. Thus the generator's power can be obtained basing on mechanical quantities (force, velocity) or electrical quantities (voltage, current). Accordingly, the flux of energy taken from the mechanical sub-system P_{mg} can be designated in two ways:

$$P_{mg} = F_g \left(\frac{dz}{dt} - v \right) = e_g i_g. \tag{9}$$

A portion of electrical power produced by the generator is converted into Joule-Lenz heat P_{gh}. The part of energy, assigned as P_{gd} is transferred to the MR damper (Figs. 4 and 5). In the model adopted here in a steady state motion the mean value of power P_{gd} transferred from the generator to the MR damper is equal to the mean value of Joule-Lenz energy flux $P_{dh}^{(e)}$. The fluxes can be obtained from the formulas:

$$P_{gh} = R_g i_g^2,$$
$$P_{dh}^{(e)} = R_d i_g^2. \tag{10}$$

In the MR damper the mean value of power P_{gd} is slightly larger than the mean value of $P_{dh}^{(e)}$. It is associated with the flux of electrical energy converted into heat in the process

of re-magnetisation of magnetic circuit elements in the MR damper and in the process of oscillatory action of the magnetic field upon the MR fluid. Ferromagnetic particles are rotated as the result of the moment of forces of the magnetic field. Next, as a result of their mutual interactions, they conglomerate to form chains. During motion, particles experience the fluid resistance and hence the work is performed. This work is converted into heat energy. In the model adopted these energies are neglected.

Internal energy of the electro-mechanical system becomes the sum of kinetic energy E_k, of potential energy E_p of elasticity and of potential energy E_e of magnetic fields in the generator's and MR damper's coils. These energies, being the functions of state, can be calculated from the formulas:

$$E_k = \frac{1}{2} m v^2,$$
$$E_p = \frac{1}{2} c (z - x)^2, \tag{11}$$
$$E_e = \frac{1}{2} (L_g + L_d) i_g^2.$$

4. Energy balance

The energy balance is based on the principle of equivalence of energy and work. In the vibration reduction system considered here, the energy flows between the mechanical and electrical sub-systems and, like in any other vibration reduction system, the energy is dissipated through its flow to the environment. Heat energy becomes the final form of dissipated energy. Conversion of mechanical to heat energy takes place directly in the mechanical sub-system as well as indirectly-via the electrical sub-system. The respective energies were defined in Sec. 3.

The energy balance involves the energy supplied to the system, the internal energy and dissipated energy. The concept of applying the power and derivatives of kinetic and potential energy was chosen in calculations. Derivatives of kinetic and potential energy can be interpreted as fluxes of internal energy forms. The energy balance is thus brought down to comparing the power supplied by the shaker with the sum of the derivative of the system's internal energy and the overall dissipated power. The power balance in the considered system can be written as:

$$P_{in} = P_{dh} + P_{gh} + \frac{dE_k}{dt} + \frac{dE_p}{dt} + \frac{dE_e}{dt}. \tag{12}$$

5. Numerical simulations

Theoretical considerations were supported by numerical simulations to find the following parameters: displacement and velocity of the body and current in the generator-MR damper circuit. They were further utilised to compute the particular energy fluxes and to establish the energy balance of the system.

The calculations were performed for the laboratory set-up outlined in [10], constructed in accordance with the diagram in Fig. 1. Parameters of this system are: $m = 100$ kg, $c = 10^5$ N/m, $R_g = 0.4$ Ω, $L_g = 7.5$ mH, $R_d = 5$ Ω,

$L_d = 100$ mH, $\kappa = 30$ N/A. For those parameters the natural frequency of the system is equal to 5 Hz. Parameters of the damper model, estimated on the basis of laboratory tests, are: $c_1 = 970$ N/A, $c_2 = 50$ N, $c_3 = 3745$ Ns/Am, $c_4 = 322$ Ns/m. These parameters are chosen such that the relationship between the damper force and velocity obtained from modelling should agree well with experimental data, for current levels in the range (0.05, 0.5) A. Experiments performed on a testing machine involved force measurements under the applied excitations in the form of preset sine displacements. A predicted and measured damper force versus velocity loop is shown in Fig. 6.
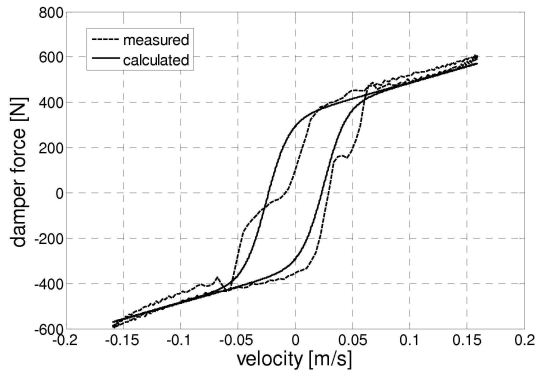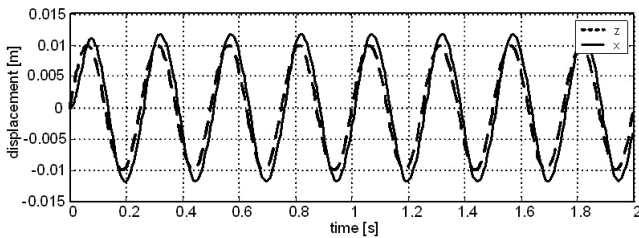

Fig. 6. Damper force versus velocity

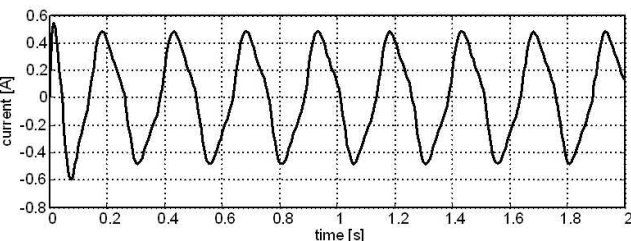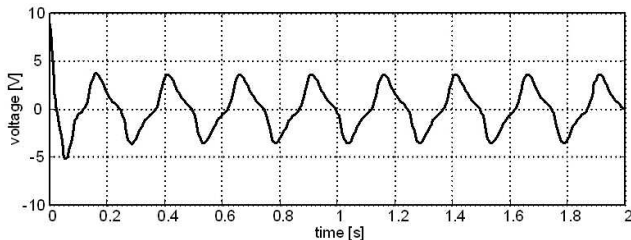Fig. 7. Displacement of the body and the kinematic excitation

Fig. 8. Voltage and current in the electrical sub-system

Simulation results for the applied kinematic excitations of frequency 4 Hz and amplitude 10 mm are shown in Figs. 7 and 8. Time patterns of kinematic displacement executed by

the shaker and of body displacement are shown in Fig. 7. The investigated system is the oscillatory one, operating under the excitation of frequency less than the natural frequency (5 Hz). The plots reveal that the gain approaches 1.2 and the phase shift equals 19°, which indicates good damping performance.

Figure 8 presents the time patterns of voltage and current in the circuit incorporating the generator and the MR damper. Plots reveal most characteristic non-symmetry between parts of curves associated with increasing and decreasing values of current and voltage. It is associated with the damper's operation profile including a hysteresis. The phase shift of current with respect to voltage is about 35°. This parameter is of great significance to ensure the effective performance of the system. Voltage is proportional to the relative velocity (Eq. (2)) and the modulus of current determines the value of the damper force (Eq. (4)). The damper efficiency is optimal when the maximal value of the damper force goes in line with extreme values of the relative velocity.

The relative velocity and the force with which the damper acts upon the body are shown in Fig. 9. The plot of damper force reveals fast changes in those times when the relative velocity is close to zero. That is the consequence of the adopted model of the MR damper.
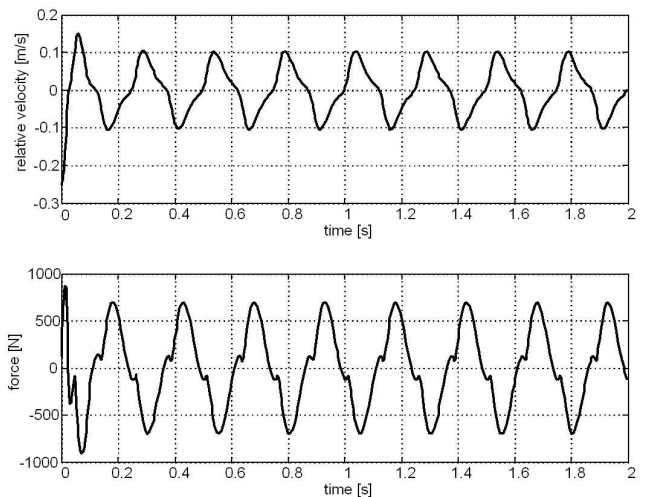

Fig. 9. Relative velocity and the damper force

As it was mentioned before, in order that a self-powered vibration reduction system should perform well, it is required that the damper force and the relative velocity of the body should be duly synchronised. In the most favourable conditions, the MR damper force should be in opposite phase to the relative velocity so that mechanical energy should be dissipated at each moment of time. In the case analysed here there are some minor, though still acceptable, phase shifts between those signals.

The calculation procedure is applied mainly to determine the energy fluxes. The results are shown in Figs. 10–12. Figure 10 presents two main fluxes of dissipated energy: that of mechanical energy P_{md} (Eq. (8)) directly converted into heat in the mechanical sub-system of the MR damper and the flux of energy converted into heat in the electrical circuit compris-

ing the generator and the MR damper $P_{gh} + P_{dh}^{(e)}$ (Fig. 5). It is reasonable to expect that the largest proportion of dissipated energy is that dispersed in the mechanical sub-system of the MR damper. The average value of power emitted by the MR damper is the result of work performed by its mechanical sub-system approaches 25 W. That is nearly 35 times the energy dissipated in the generator-MR damper circuit.
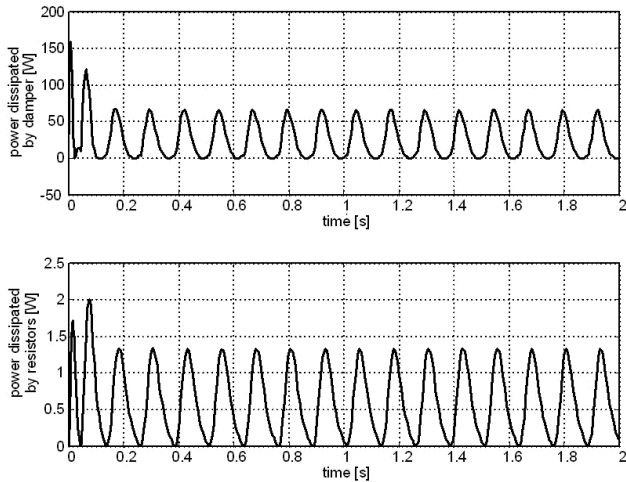

Fig. 10. Power output directly from the mechanical sub-system P_{md} and via the electrical sub-system $P_{gh} + P_{dh}^{(e)}$

Figure 11 shows the derivatives of kinetic and potential energy and the energy of magnetic fields placed in coils in the generator-MR damper circuit. It appears that kinetic energy of the system tends to vary significantly. Minor variations of potential energy of elasticity are associated with small relative displacements of the body. Variations of the magnetic field energy are decidedly smaller. A reason is associated with small relative velocity. For excitation frequencies in excess of resonance frequency, the proportion of potential energy variations tends to increase.
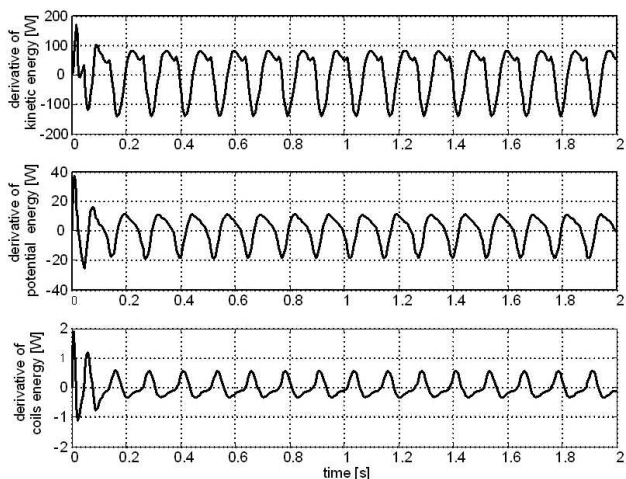

Fig. 11. Derivatives of kinetic energy, potential energy and coils energy

The energy balance of the system (Eq. (12)) is illustrated in Fig. 12. The first plot shows power supplied to the system (left-hand side of Eq. (12)); the other plot gives the sum of dissipated powers and derivatives of particular forms of internal energy (right-hand side of Eq. (12)). Power supplied and the sum of dissipated power and derivatives of particular forms of internal energy were determined separately. The difference between these quantities is associated with numerical errors. Comparison of the two plots suggests good accuracy of calculation results.
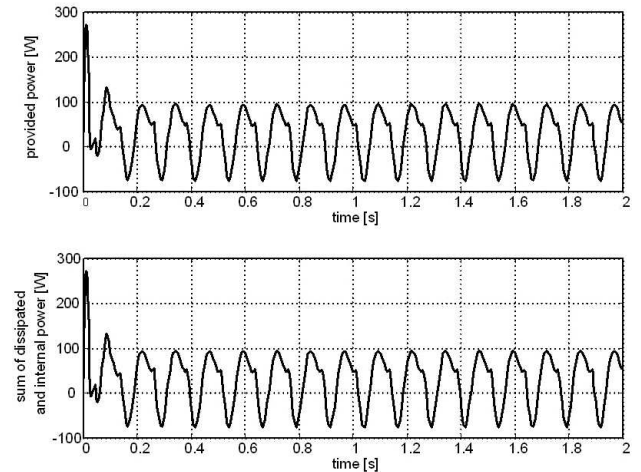

Fig. 12. Supplied power and sum of dissipated power and the derivative of internal energy

6. Summary

The vibration reduction system considered in the study is a self-powered system. Energy required to power-supply the MR damper is recovered from the vibrations. An effective design of such systems requires a thorough analysis of matter flow.

The adopted model of the system allows for estimating the flow of power in the considered system. The model is based on the linear model of a generator and a nonlinear model of the MR damper. The calculation procedure yields the power input to the system via the shaker and power dissipated in the mechanical and electrical sub-systems.

Calculation results lead us to the following conclusions.

The generator employed in the system acts as a source of electrical energy for the MR damper. The required energy is absorbed from the vibrations. Energy obtained by the generator accounts for about 2.5% of energy dissipated by the MR damper in its mechanical sub-system. In the context of a mechanical system, the generator acts as another damper dissipating energy. However, its contribution to the damping of the body's vibrations is minor.

Energy recovered by the generator from the mechanical sub-system is partly converted into heat in the generator windings, whilst the remaining part is utilised in the MR damper. This portion of recovered energy can be treated as energy effectively used by the MR damper.

The main portion of energy is dissipated in the consequence of the flow resistance of the MR fluid during the piston motion with respect to the MR damper housing. The magnitude of this energy flux is controlled by a small flux of energy

expended to arrange the magnetic particles in the MR fluid in the required configuration. This property consisting in control of energy dissipation by small flux of energy is typical for semi-active 'energy-saving' systems.

Research is now being carried out to verify experimentally the results obtained in numerical simulations.

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