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Theory and experimental verification of thermal stresses in Cosserat medium

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Abstract. An asymmetrical inter-atomic action, basic for a thermal couple stress presence, has been investigated in this work. A mechanical description of a Cosserat thermoelasticity is replaced by an equivalent magnetic representation. Then, a magnetization of a medium is analyzed. An experimental demonstration of the occurrence of the non-central load of the atom has been made applying the Electron Paramagnetic Resonance spectrometer. The mathematical model of the measurements has been presented in form of Bloch equations with extension to Cosserat medium.

Key words: EPR method, nonsymmetrical thermoelasticity.

1. Introduction

The hypothesis of the couple stresses [1, 2], introduced into the thermoelasticity [3–6], has not been experimentally verified yet. The way of determining six Cosserat elastic constants is shown in [7]. The physical phenomenon (nonlinear birefringence) of an experimental analysis of Cosserat medium has been proposed in [8]. The mechanism of the couple stress formation in nano-scale has been analyzed in the presented work. The theoretical description is verified by the magnetic resonance method [9]. The presented work is an interpretation of the Cosserat birefringence [8] in nano-scale.

Motion at molecular level and at intermolecular action in nanomechanics are modelled with a help of the principles of the classical mechanics applied to the systems of the material points. The study will be quite different when the model of the material points is replaced by "nanocosmos" of the electrons cloud. From this point of view and taking into account the Fracture Mechanics [10], Nanomechanics of Materials [11], Materials Engineering, Dislocations Theory, adhesion, friction, phase transition, etc. we focus on new phenomena which are essential in nanoengineering and which determine the behaviour of materials in micro, meso and macro scales. In this work we try to explain the phenomenon of the couple stresses problem including the atomistic nature of the nanomatter.

2. Cosserat medium

The Cosserat medium is defined as a crystalline body. The inter-atomic actions $\boldsymbol{P_{therm}}$ are nonsymmetrical Fig. 1. The nonsymmetrical actions generate the moment of actions $\boldsymbol{M_{couple}}$ which causes a disturbance of the electrons move, Fig. 1. An additional move of the electrons creates an additional magnetic phenomenon which is combined with couple stress.

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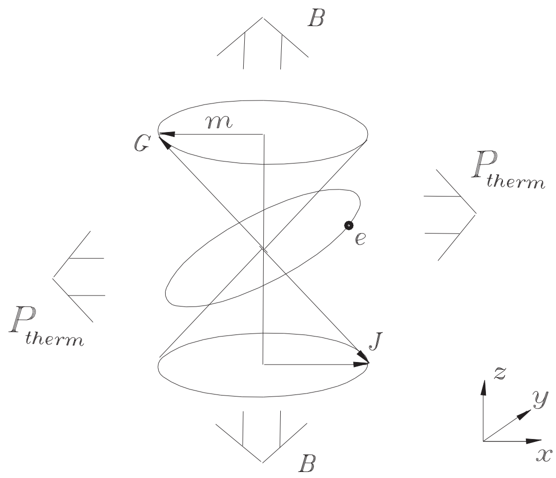

Fig. 1. Precession of the vectors of the angular moment $\boldsymbol{J}$ and magnetic moment $\boldsymbol{G}$ in non-central thermal field $\boldsymbol{P_{therm}}$

2.1. Couple nano-moment. The thermal load of the material causes the inter-atomic action: $P_{therm} = -\beta\Delta\left(\delta\left\langle r\right\rangle\right)$, β represents elasticity constant, $\Delta\left(\delta\left\langle r\right\rangle\right)$ is difference between the increases of the deflection mean $\delta\left\langle r\right\rangle$ of the atoms from the equilibrium positions as a result of the temperature increase. The nonsymmetrical action $\boldsymbol{P_{therm}}$ approaches to the placing of the vector of the angular momentum $\boldsymbol{J}$ of the atom in the perpendicular position to $\boldsymbol{P_{therm}}$. This loading system causes the precession $\boldsymbol{\omega_{couple}}$ of the electron orbit around the direction perpendicular to $\boldsymbol{P_{therm}}$.

The couple nano-moment is described by the following formula:

$$\boldsymbol{M_{couple}} = \boldsymbol{\omega_{couple}} \times \boldsymbol{J}. \tag{1}$$

The move of the atom in the form of precession is recognized as an elementary factor for arising thermal couple stresses. The precession of the electron generates the magnetic fields. This additionally magnetic field has an induction

$\boldsymbol{B^{couple}}$. Now the couple nano-moment can be written in the form:

$$\boldsymbol{M_{couple}} = \boldsymbol{p_m} \times \boldsymbol{B^{couple}}, \tag{2}$$

where: $\boldsymbol{p_m} = -\gamma \boldsymbol{J}$ is a magnetic moment of the atom, γ is a magneto-mechanical ratio. For the elementary unit of the volume V, the vector of magnetization $\boldsymbol{G^{\mu}}$ $(G_x^{\mu}, G_y^{\mu}, G_z^{\mu})$ according to the magnetic field $\boldsymbol{B^{\mu}} = \sum_1^N \left(\sum \boldsymbol{B_i^{couple}} \right)_k$ is generated by N atoms, when Z is atomic number, i is the number of the electrons, k is the number of the atoms. This magnetization $\boldsymbol{G^{\mu}}$ is derived from the couple moment of the thermoelasticity as a result of the non-central inter-atomic action.

3. Cosserat medium in resonance cavity

3.1. The Bloch equation extension. In the EPR spectrometer cavity there is a sample with the thermal stresses which are built in it. This sample is under the action of the magnetic field: $\boldsymbol{B^{EPR}} = B_1 \left(\vec{\boldsymbol{i}} \cos \omega t - \vec{\boldsymbol{j}} \sin \omega t \right) + B_o \vec{\boldsymbol{k}}$, where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors parallel to x, y, z axes, Fig. 1, ω is frequency of the lateral magnetic field, t is time. The magnetic action in the EPR spectrometer is described by the Bloch equations [12]. Additionally, when the thermal load is considered, the Bloch equations can be written in the following form:

$$\frac{d\boldsymbol{G}_x}{dt} = \gamma \left[\boldsymbol{G} \times \left(\boldsymbol{B^{\mu}} + \boldsymbol{B^{EPR}} \right) \right]_x + \frac{G_x^{\mu} - G_x}{T_x}, \tag{3}$$

$$\frac{d\boldsymbol{G}_y}{dt} = \gamma \left[\boldsymbol{G} \times \left(\boldsymbol{B^{\mu}} + \boldsymbol{B^{EPR}} \right) \right]_y + \frac{G_y^{\mu} - G_y}{T_y}, \tag{4}$$

$$\frac{d\boldsymbol{G}_z}{dt} = \gamma \left[\boldsymbol{G} \times \left(\boldsymbol{B^{\mu}} + \boldsymbol{B^{EPR}} \right) \right]_z + \frac{G_z^{\mu} + G_o - G_z}{T_z}, \tag{5}$$

where T_x, T_y, T_z are the relaxation times of the interaction between the magnetic field $\boldsymbol{B^{EPR}}$ and the stressed crystal lattice according to x, y, z axes, G_o is a z-component of the magnetization which comes from z-component of the EPR magnetic field $B_o\vec{\boldsymbol{k}}$.

After operations and substitutions: $\omega_o = \gamma B_o$, $\omega_1 = \gamma B_1$, Eqs. (3–5) can be written in the forms:

$$\begin{aligned} \frac{dG_x}{dt} &= \left(\omega_z^{\mu} + \omega_o \right) G_y - \left(\omega_y^{\mu} - \omega_1 \sin \omega t \right) G_z \\ &\quad + \frac{G_x^{\mu} - G_x}{T_x}, \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{dG_y}{dt} &= -\left(\omega_z^{\mu} + \omega_o \right) G_x + \left(\omega_x^{\mu} + \omega_1 \cos \omega t \right) G_z \\ &\quad + \frac{G_y^{\mu} - G_y}{T_y}, \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{dG_z}{dt} &= \left(\omega_y^{\mu} - \omega_1 \sin \omega t \right) G_x - \left(\omega_x^{\mu} + \omega_1 \cos \omega t \right) G_y \\ &\quad + \frac{G_z^{\mu} + G_o - G_z}{T_z}. \end{aligned} \tag{8}$$

where $\omega^{\mu} \left(\omega_x^{\mu}, \omega_y^{\mu}, \omega_z^{\mu} \right)$ is precession of the magnetization of the vector $\boldsymbol{G}^{\mu}$ under the thermal load action.

3.2. Hydrostatic state. The hydrostatic state of the Cosserat thermoelasticity is described as follows:

$$B_x^{\mu} = B_y^{\mu} = B_z^{\mu} = B^{\mu}, \tag{9}$$

$$G_x^{\mu} = G_y^{\mu} = G_z^{\mu} = G^{\mu}, \tag{10}$$

$$\omega_x^{\mu} = \omega_y^{\mu} = \omega_z^{\mu} = \omega^{\mu} = \gamma B^{\mu}, \tag{11}$$

For values of the B_I, B^{μ} not large when compared to B_o and after the substitution: $T_z = T_1$, $T_x = T_y = T_2$, the solution of Eqs. (6)–(8) is obtained in the form:

$$G_x = m \cos (\omega t + \varphi) + G^{\mu}, \tag{12}$$

$$G_y = -m \sin (\omega t + \varphi) + G^{\mu}, \tag{13}$$

$$G_z = G_o + G^{\mu}, \tag{14}$$

where:

$$m = \frac{\left[G_o \left(\omega_1 + \omega^{\mu} \right) - G^{\mu} \left(\omega_o - \omega_1 \right) \right] T_2}{\sqrt{1 + \left[\left(\omega_o + \omega^{\mu} \right) - \omega \right]^2 T_2^2}}, \tag{15}$$

$$tg\varphi = \frac{1}{\left[\left(\omega_o + \omega^{\mu} \right) - \omega \right] T_2}, \tag{16}$$

m is the amplitude of the lateral magnetization, φ is the azimuth angle between $\boldsymbol{G}$ and $\boldsymbol{B}$.

3.3. Absorptivity. The rate of energy absorption by the system of the atoms in a cavity of the EPR spectrometer is described by the formula:

$$N = \boldsymbol{B^{EPR}} \frac{d\boldsymbol{G}}{dt}. \tag{17}$$

Substituting components of $\boldsymbol{B^{EPR}}$ and components of the magnetization $\boldsymbol{G}(G_x, G_y G_z)$ the following formula is obtained:

$$N = \frac{\omega B_1 T_2 \left[G_o \left(\omega_1 + \omega^{\mu} \right) - G^{\mu} \left(\omega_o - \omega_1 \right) \right]}{1 + \left[\left(\omega_o + \omega^{\mu} \right) - \omega \right]^2 T_2^2}. \tag{18}$$

Basing on the derivation formula (18) it is expected that an asymmetrical inter-atomic action causes a change of the absorptivity in EPR spectrometer relatively to the material without thermal load when $\omega^{\mu} = 0$, $G^{\mu} = 0$. The expected phenomenon is also the experimental proof of the presence of the couple stresses in the loaded material.

4. Measurement

The crystals of the hydrated copper sulfate $C_uSO_4{\cdot}5H_2O$ have been studied. The dimensions of the crystals do not exceed 2 [mm] measured at the diameter of the sphere circumscribed to the crystals. The uniform compression of the samples was made by setting the crystals in Epoxy E51 cured by butyl phthalate in the quantity of 10 weight parts per 100 Epoxy E51 weight parts. The samples were compressed by the chemical and the thermal contraction. So, asymmetrical interatomic action is described by formula: $P_{therm} = -\beta r_o \Delta T \left(\alpha_1 - \alpha_2 \right)$, where α_1, α_2 are the coefficients of the thermal expansions of the $C_uSO_4{\cdot}5H_2O$ and Epoxy E51 respectively, r_o is equilibrium position of the atom, ΔT is the increase of temperature. The spectra for free crystals and then for the same crystals placed in the Epoxy were made. The experiments were carried out at room temperature and at temperature of liquid nitrogen. The Electron Paramagnetic Resonance spectra were measured with spectrometer ELEXYS 500 produced by Bruker (Karlsrue) at frequency band X (9.5 GHz). EPR spectra were recorded by the amplitude of the modulation 5 [mT] and at the power of the microwave 10 [mW].

The crystal field in the copper sulfate leads to the effect of the extinction of the orbital angular moment, so the magnetic moment is derived from electron spin only [12].

The EPR spectra were recorded in the form of the first derivative of the power absorption curves. The exemplary results for one of the sample are presented in Figs. 2, 3.

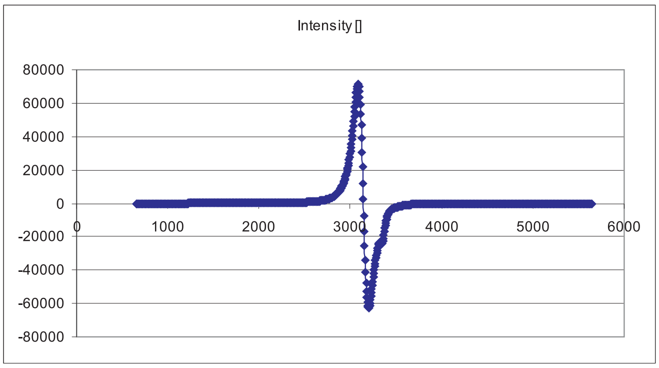

Fig. 2. EPR absorption spectrum for free crystal

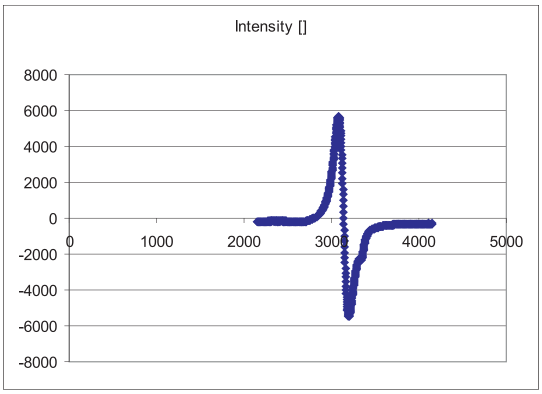

Fig. 3. EPR absorption spectrum for thermal loaded of the crystal

The intensity of absorptivity of the Epoxy E51 was measured separately. It was noted that it appeared to be some orders lower, whereby it did not influence the results.

Available parameters:

Averaged Scans	1
Resonator	st 9733
State of Aggregation	C
Sampling time [s]	0.16384
Field Mod. Amplitude	0.0005
Field Mod. Frequency [Hz]	100000
Microwave Frequency [Hz]	9.75586×10^9 – free crystal, 9.74121×10^9 – loaded crystal
Microwave Power [W]	0.0100825
Receiver Gain	20
Receiver Time Constant [s]	0.16384
Receiver phase [deg]	0.0
Receiver Harmonic	1
Receiver Offset [%FS]	0.0

Axis Range:

Field [G]	650 to 5650, 1024 pts width 5000, center 3150

Measurement parameter:

Intensity of absorptivity [unit]	134 000 – for free crystal 11 104 – for loaded crystal

5. Results

We obtained the intensities of the power absorption by the system of atoms for a free crystal: 134000 [unit], Fig. 1, and for the crystal under the action of the thermal loading: 11104 [unit], Fig. 2. The difference is 122896 [unit]. We explain this difference by magnetization caused by non-central action between the atoms during thermal loading of the crystal. This result is in conformity with the expectation that is described by the formula (18). The rate of energy absorption for the loaded crystal is lessen in relative to a free crystal which can also be expected from formula (18).

6. Conclusions

This analysis proves that the hypothesis of the couple thermal stresses has been experimentally verified in form of the physical phenomena of precession of electron orbits and magnetization of the Cosserat medium.

It is well-founded to introduce the intermolecular moment action and rotation of molecule for analysis of the problems of the nonsymmetrical thermoelasticity [3–6] and thermal couple stress nanomechanics [13].

The phenomena of the magnetization of the Cosserats medium in the field of the thermal load give us possibilities of experimental study of the couple thermal stresses by the methods of the magnetic resonance (EPR, NMR,. . .) [9].

To interpret the EPR spectra we have applied Bloch equations extended to the Cosserat medium.

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