BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES
Vol. 56, No. 4, 2008

Parallel compensator versus Smith predictor for control of the plants with delay

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Abstract. In the paper the parallel compensator considered in [1] is applied to control of the plants with delay and compared with Smith predictor. It is noted that Smith predictor removes only the delay, while the parallel compensator also changes the dynamics of the replacement plant; the latter may be in some degree of freedom shaped by the designer. Owing to this the transients of the system with parallel compensator are significantly faster. Accounting implementability, the control saturations are introduced in control algorithms. Additionally it is shown that using parallel compensator we may apply a relay control to the plants with delay and nonminimum phase zeros, which is impossible by using Smith predictor. Several results of simulations are described which confirm these observations.

Key words: parallel compensator, Smith predictor, plants with delay, nonminimum phase plants.

1. Introduction

The plants with pure time delay belong to the so called difficult plants, for which it is difficult to design a regulator assuring some appropriate accuracy of the control. For these plants an insignificant increase of the proportional regulator gain causes instability and for small gain the closed loop system has unsatisfactory accuracy in steady state. Also because of the demand for stability of the closed loop system, the integral part of the regulator must have a small gain. This part removes the steady state error, but because of the small gain the system is very slow. An exhaustive elaboration of the problems related with the systems with delay one may find in the monograph of Górecki [2].

For the plants with pure time delay Smith [3] proposed a compensator which effectively takes the delay outside the loop and allows a feedback design based on the plant dynamics without delay. The result is that the regulator designed in this manner is faster and assures higher accuracy (in comparison to the system in which the delay remains in the loop). Now this compensator is commonly called Smith compensator [4] (or predictor [5]).

The idea of the parallel compensator for the systems with so called difficult plants (with delay, and/or nonminimum phase and/or of higher order) was considered in [1], by the author of the present paper. The parallel compensator connected in parallel to the plant changes the dynamics of the latter, so that the obtained replacement plant is easier for control.

Another approach to parallel compensator was presented in [6] and in its references, where the plants with structured uncertainty were considered. In comparison to [1] and the present paper the considerations of [6] are significantly more complicated, though they concern only minimum phase plants and are not related with the present paper.

In the present paper the system with parallel compensator considered in [1] is used for the systems with delay and compared with the system with Smith predictor. It is noted that the parallel compensator not only removes the delay from the resulting replacement plant, but also changes the dynamics of the latter, significantly. The Smith predictor removes only the delay from the resulting replacement plant, but except this the dynamics of the latter remains unchanged. The result is that the system with parallel compensator may have significantly faster transients which has been confirmed by many performed simulations.

It is noted that the parallel compensator applied for the control of the plants with delay and nonminimum phase zeros, gives possibility of applying a relay control; the resulting system may be treated as modified sliding mode control. Relay control may be interesting for some users because of simplicity of the actuator. At the same time it is noted that Smith predictor does not make it possible to implement the sliding mode relay control for the plants with delay and nonminimum phase zeros.

2. Parallel compensator

In this section the idea of the parallel compensator introduced in [1] is reminded.

Consider the linear plant described by the transfer function (TF)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{L(s)}{M(s)} e^{-s\tau}, \tag{1}$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the plant output and input, respectively, while $L(s)$ and $M(s)$ are polynomials of m-th and n-th degree, respectively, $m < n$, τ is the time of delay. Assume that the plant is stable, that is its poles p_i, $i = 1, 2, ..., n$ have negative real parts i.e. $Rep_i < 0$.

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In the case of difficult plant (e.g nonminimum phase, and/or with higher order dynamics, as well as with pure time delay), when it is difficult to design the regulator assuring an appropriate accuracy, the parallel compensator shown in Fig. 1a may be applied. The parallel compensator is described by the TF

$$G_c(s) = \frac{Y_c(s)}{U(s)} = G_1(s) - G(s), \tag{2}$$

and its idea, as it was noted in [1] is similar to that of the Smith predictor. Here $Y_c(s)$ is the Laplace transform of the output y_c of the compensator, while $G_1(s)$ is the TF which will be appropriately chosen.

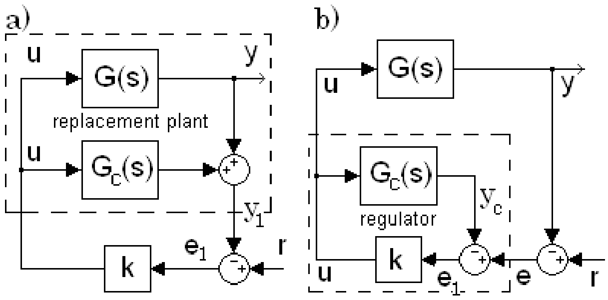

Fig. 1. The equivalent block diagrams of the system with parallel compensator

Note that in the proposed structure shown in Fig. 1a the TF of the replacement plant outlined by the dashed line is described by

$$\begin{aligned} \frac{Y_1(s)}{U(s)} &= G(s) + G_c(s) \\ &= G(s) + G_1(s) - G(s) = G_1(s). \end{aligned} \tag{3}$$

Of course, to implement a closed loop (CL) stable system with the reference signal r determining the demanded output y the TF $G_1(s)$ should fulfill some demands.

In the case of regulation when $r =$ const the error in a constant steady state is mainly interesting, therefore for some constant steady state values it should be

$$y_c = 0, \quad y_1 = y, \quad e_1 = r - y_1 = r - y. \tag{4}$$

The latter condition will be fulfilled if

$$G_1(0) = G(0) = k_p, \tag{5}$$

where k_p is the gain of the plant. In the case of tracking of the varying reference signal r with the frequencies ω belonging to some working frequency band

$$\omega \in [0, \omega_{mx}], \tag{6}$$

the demand (4) should be at least approximately fulfilled for frequencies (6),

$$G_1(j\omega) \approx G(\omega) \quad \text{for} \quad \omega \in [0, \omega_{mx}]. \tag{7}$$

The further considerations are limited to the case of regulation.

As it results from Fig. 1a, our considerations are also limited to the case when a proportional P regulator is used with high gain k. The CL system with the replacement plant $G_1(s)$ and high gain k should be stable and should have some appropriate phase margin $\Delta\varphi_1$. It will be shown that the latter demand may be fulfilled if the TF $G_1(s)$ has the relative degree equal to one, and its parameters are appropriately chosen.

3. Approximate description of the CL system

The equivalent block diagram of the system from Fig. 1a is shown in Fig. 1b. Note that the part of the system outlined by the dashed line contains the elements of the regulator based on the parallel compensator. Assuming that the system has appropriate phase margin, under high gain k, the regulator in the system is described by the following TF

$$C(s) = \frac{U(s)}{E(s)} = \frac{k}{1 + kG_c(s)} \approx \frac{1}{G_c(s)}. \tag{8}$$

Accounting (8) we obtain the following formula describing the CL system

$$\frac{Y(s)}{R(s)} = \frac{G(s)/G_c(s)}{1 + G(s)/G_c(s)} = \frac{G(s)}{G_c(s) + G(s)} = \frac{G(s)}{G_1(s)}. \tag{9}$$

The formula (9) may be used for designing the TF $G_1(s)$. This will be discribed in the next section.

4. Design of the replacement plant transfer function

Denote by

$$G_1(s) = \frac{L_1(s)}{M_1(s)}, \tag{10}$$

a stable replacement plant TF (3) with minimum phase zeros. Thus the polynomials $L_1(s)$ and $M_1(s)$ are Hurwitz polynomials. One way of designing $G_1(s)$ is to choose

$$M_1(s) = M(s), \tag{11}$$

$$L_1(s) = l(1 + sT)^{n-1}, \quad l = L(0), \tag{12}$$

so the condition (5) is fulfilled.

Denote by $\varphi_1(\omega)$ the phase of the frequency response $G_1(j\omega) = L_1(j\omega)/M(j\omega)$. Let the phase $\varphi_1(\omega)$ fulfills the inequality

$$\varphi < \varphi_1(\omega) \leq 0, \tag{13}$$

where $\varphi = -180°$.

Since the TF $G_1(s)$ has the relative degree equal to one then

$$lim_{\omega \to \infty} \varphi_1(\omega) = -90°. \tag{14}$$

Accounting (13) and the fact that $G_1(0) = k_p$ one may note that

$$\Delta arg_{-\infty < \omega < \infty}[1 + kG_1(j\omega)] = 0, \tag{15}$$

for any $k > 0$. The condition (15) determines the Nyquist stability criterion for the system shown in Fig. 1 [7]. Since the criterion is fulfilled then the system is stable for any $k > 0$. Additionally from (14) it results that for sufficiently large k the phase margin $\Delta\varphi_1$ is close to 90°, which results from (14) and from definition of phase margin.

Accounting (1), (10), (11) in (9) we obtain for the CL system

$$\frac{Y(s)}{R(s)} = \frac{L(s)}{L_1(s)} e^{-s\tau}. \tag{16}$$

From these considerations it results that in the considered case the choice of $L_1(s)$ influences the dynamics of the researched CL system, essentially. Really, the characteristic equation of the CL system is

$$L_1(s) = 0, \tag{17}$$

and its roots influence the velocity of decay of the transient response. Therefore we try to choose $L_1(s)$ in the form (12) containing the multiple root $s_i = -1/T, i = 1, 2, ...n-1$. Of course, to obtain fast transient, we should choose a possibly small time constant T, for which the condition (13) is fulfilled. For the chosen T the condition (13) may be easy checked using MATLAB command `nichols(.)` (or `nyquist(.)`).

There arises the question when the choice of $L_1(s)$ in the form (12) fulfilling (13) is possible. One may suppose that it is possible if the dynamics of particular modes of denominator $M(s)$ of (1) is comparable (i.e. there are no modes with significantly faster or significantly slower dynamics; significantly means several or more times faster or slower).

If it impossible to find $L_1(s)$ in the form (12) fulfilling (13) then we must seek it among other reasonable forms giving possibly fast transient. Some help in seeking is the observation that any mode of the type $(1+sT_i)$ or $(1+\alpha_1 s+\alpha_2 s^2)$ corresponding to real and complex conjugate roots, respectively, appearing in denominator or numerator TF gives decrease of the phase $\varphi_1(\omega)$ by $-90°$ or $-180°$ or increase by $90°$ or $180°$ when ω is varying from 0 to ∞, respectively. Taking this into account the form of the possibly fast $L_1(s)$ may be chosen for which condition (3) is fulfilled.

If for instance one mode of the denominator $M(s)$ is significantly slower then the remaining modes then $L_1(s)$ may be created from the remaining faster modes of the denominator. The result is that the TF $G_1(s)$ takes then the form of the first order lag containing in denominator the slowest not cancelled mode (the faster modes of the denominator $M(s)$ and numerator $L_1(s)$ as stable, may be cancelled).

4.1. Example 1. Consider the plant described by the following TF

$$G(s) = \alpha \frac{-0.5s+1}{s^4+3.5s^3+6s^2+7s+3} e^{-s\tau}. \tag{18}$$

The TF $G(s)$ has the following stable poles $p_1 = -1.8185$, $p_2 = -0.7343$, $p_3 = -0.4736 + j1.4221$, $p_4 = -0.4736 - j1.4221$ and the one nonminimum phase zero $z_1 = 2$, while $\alpha = 1$ and $\tau = 1$. The plant is of forth order with delay and nonminimum phase.

To design the parallel compensator we choose the TF $G_1(s)$ in the form determined by (10–12). Time constant T has been chosen after several trials with using `nichols(.)` MATLAB command. From these trials it results that for $T = 0.3$ the minimal phase of $G_1(j\omega)$ is equal to $-167°$ and the condition (13) is fulfilled with some margin. Accounting (12) with $T = 0.3$ we obtain

$$G_1(s) = \frac{0.027s^3+0.27s^2+0.9s+1}{s^4+3.5s^3+6s^2+7s+3}. \tag{19}$$

The parallel compensator is determined by formula (2) together with (18) and (19). The gain $k = 1000$ has been chosen to obtain 0.3% accuracy in steady state. Change of the value of the gain from $k = 300$ to $k = 3000$ and more practically do not change the results of simulations (except the steady state error).

The results of simulations for reference value $r = \mathbf{1}(t-1)$ ($\mathbf{1(t)} = \mathbf{0}$ for $t < 0$ and $\mathbf{1(t)} = \mathbf{1}$ for $t > 0$) are shown in Fig. 2. The undershot equal to 0.25 appears in the time response of y, which usually appears in nonminimum phase systems. It is seen that the steady state value of the output y is achieved for $t_1 \approx 4.5$ (with 1% accuracy) and the transient lasts approximately 4.5 units of time. At the same time at $t \approx 1$ and $t \approx 1.15$ the peaks $u \approx 1000$ and $u \approx -140$ appear, respectively, in the control signal, which is the result of the high gain k of the regulator based on the parallel compensator. Such a high values of the control signal may be non-implementable in practice, therefore in the following considerations the saturations of the control variable u are introduced.

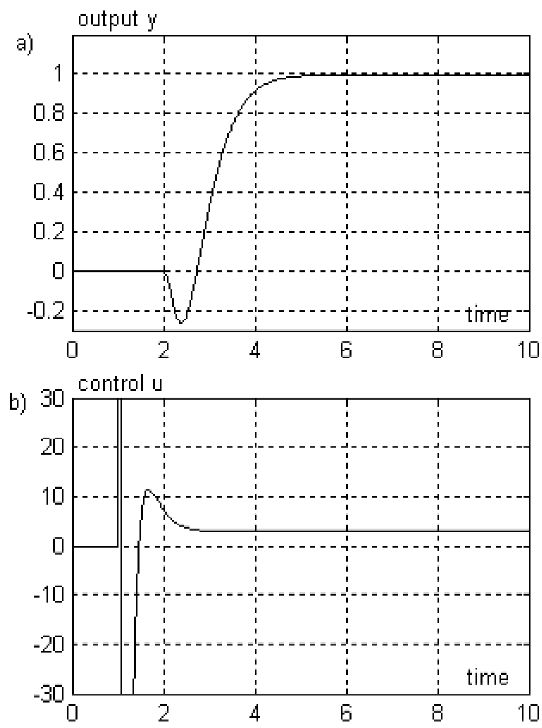

Fig. 2. a) The time response of the output y and b) of the control u – for the system from Example 1

It is worthwhile to add that the system is robust. The increase of the coefficient α to $\alpha = 1.5$ without change of the parallel compensator gives an output response y with acceptable overshot. For $\alpha = 2$ in the output response there appear oscillations but the system remains stable. Decrease of α even to $\alpha = 0.25$ increases only the settling time but the system responses are acceptable.

5. Accounting saturation of the control

Since the approximate TF (8) of the regulator has high gain k, the response of the regulator to a stepwise change of the reference or output disturbance signals may contain high value peaks which in practice may be non-implementable. To account limitations of the control signal the nonlinearity accounting saturations has been introduced in the block diagrams in Fig. 3a and b.

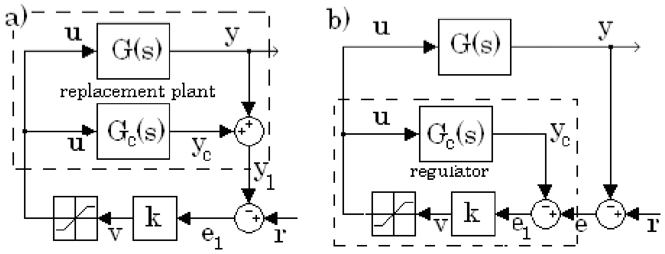

Fig. 3. The equivalent block diagrams with accounting control saturation and outlining a) the replacement plant, b) the resulting regulator

The nonlinearity accounting saturations is described by the usual formulas

$$u = \begin{cases} u_{mx} & \text{for} \quad v \geq u_{mx} \\ v & \text{for} \quad u_{\min} \leq v \leq u_{mx} \\ u_{\min} & \text{for} \quad v \leq u_{\min} \end{cases} \tag{20}$$

where u_{mx} and $u_{\min}$ denote the upper and lower saturation.

Note that the block diagram from Fig. 3a contains the same replacement plant described by the TF (3) fulfilling (13) and additionally contains the saturation block. To obtain the equivalent block diagram shown in Fig. 3b the saturation block must be introduced to the resulting outlined regulator which in this case becomes nonlinear.

More convenient for analyzing stability is the block diagram from Fig. 3a. To analyze the possibility of appearance of stable oscillations (i.e stable limit cycle) the describing function analysis may be applied. Assume that $u_{\min} = -u_{mx}$ i.e the function (20) describing saturation is odd. Let $N(A)$ is the describing function for this saturation (A is the amplitude of the sinusoidal input v) [8]. Then it is $0 < N(A) \leq 1$ for $0 < A < \infty$ and the negative inverse $-1/N(A)$ on the Nyquist complex plain occupies the negative part of the real axis lying to the left from the point $[-1, j0]$ ($j = \sqrt{-1}$) and the point $[-1, j0]$. Note that if the $G_1(j\omega)$ fulfills the condition (13) then for any even very high value of k there is no intersection of the frequency response $kG_1(j\omega)$ with the locus $-1/N(A)$, $0 < A < \infty$. Thus in accordance with the describing function analysis the system from Fig. 3a (and 3b) should be stable. Therefore we may suppose that the existence of saturation in the system may only decrease the speed of returning to steady state and temporary high errors e. The performed simulations confirm this observation.

Note, that if for some frequency ω^* $\varphi_1(\omega^*)$ is close to $-180°$ (i.e. is close to the locus $-1/N(A)$), but fulfills the condition (13), then in the system may exist the slowly decaying oscillations with the frequency close to ω^*, which may increase the time of returning to steady state. Therefore in the case of introduction of the saturation (20) it may be save to increase the lower border φ in the condition (13) (e.g. it should be $\varphi = -150°$).

5.1. Example 2. Now consider the system shown in Fig. 3 with the plant TF (18), parallel compensator TF (2) with (19) and (18), gain $k = 1000$ and with introduced control saturations determined by $u_{\min} = -20$ and $u_{mx} = 20$. The time response of the CL system to the reference value $r = \mathbf{1}(t-1)$ is shown in Fig. 4. It is seen that though in the initial period of time the control u is different from that shown in Fig. 2b, the plot of y has been changed in-significantly (compare Figs. 4a and 2a). Really it has somewhat smaller undershot equal to 0.2 but the time t_1 is almost the same as for the system without saturations considered in Example 1 (strictly speaking it is in-significantly greater). Thus the appearance of the not to small control saturations influences the time response in-significantly. Even for smaller values of the saturations (e.g. $u_{\min} = -10$ and $u_{mx} = 10$) the increase of t_1 is insignificant, but there appear some exactly shown deformations of the transient.

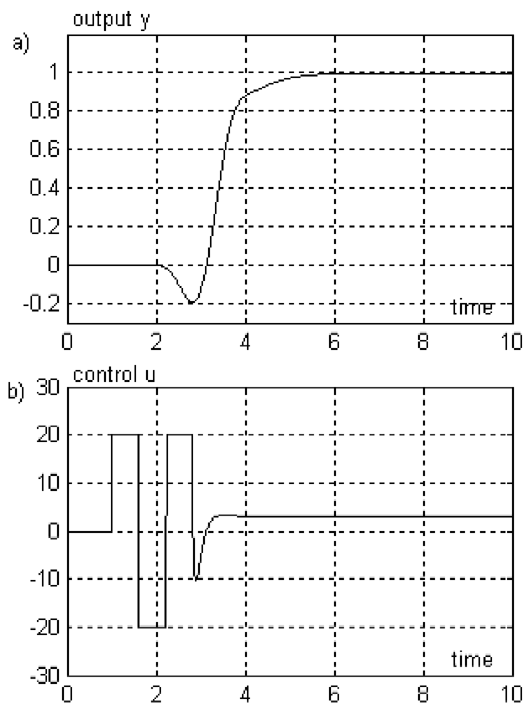

Fig. 4. a) The time response of the output y and b) of the control u – for system from Example 2

6. Applying of Smith predictor

Now, consider the system shown in Fig. 6, in which in the place of parallel compensator it appears the Smith predictor described by the TF [3]

$$G_S(s) = \frac{L(s)}{M(s)}(1 - e^{-s\tau}), \qquad (21)$$

then the replacement plant TF takes the form

$$G_{1S}(s) = \frac{L(s)}{M(s)}. \qquad (22)$$

Comparing the dynamics of the plant $G(s)$ (1) and of the replacement plant $G_{1S}(s)$ (22) we see, that in $G_{1S}(s)$ only the delay has been removed. Except this, the dynamics of $G(s)$ is the same as that of $G_{1S}(s)$. Thus if $G(s)$, beyond the delay contains also some nonminimum phase zeros, then these zeros appear also in $G_{1S}(s)$ and the latter plant is also difficult to control.

Another situation is when in the place of Smith predictor the parallel compensator is applied, since then the replacement plant dynamics $G_1(s)$ may be shaped by the designer. To make a comparison with the results obtained in the Examples 1 and 2, in the next example for the plant $G(s)$ (18) the Smith predictor together with regulator PID will be applied.

6.1. Example 3. Now consider the classical PID regulator described by the TF

$$C(s) = k_1 + \frac{k_2}{s} + k_3\frac{s}{1 + T_d s}, \qquad (23)$$

which is applied to control of the plant (18) with Smith predictor, as shown in Fig. 5. After trials the parameters of the regulator (23) have been tuned to the values $k_1 = 1.3728$, $k_2 = 1.0212$, $k_3 = 0.3333$, $T_d = 1/300$. To obtain comparable conditions of operation, the saturation of the control u on the levels $u_{\min} = -20$ and $u_{mx} = 20$ has been introduced in simulations. The time response of the system for the reference value $r = \mathbf{1}(t-1)$ is shown in Fig. 6. It is seen that the obtained response is significantly slower than that shown in Fig. 4a. The steady state of the output is now achieved (with 1% accuracy) for $t_1 \approx 13.3$ time units. Thus the period of appearance of transients is now about three times longer than for the system with parallel compensator.

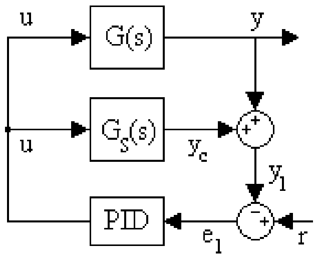

Fig. 5. The system with Smith predictor and classical PID regulator

From many trials it results that it is impossible to obtain a smaller time t_1. It was also observed that the influence of the assumed values of saturations on the time response is negligible.

One may note that the undershot appearing in the system with Smith predictor and PID regulator is significantly smaller than that in the system with parallel compensator. This is related with weaker reaction of the control in the system with PID regulator. The system with parallel compensator has a stronger reaction of the control in some initial period of time, owing to this the period of transient is shorter, but the undershot is higher.

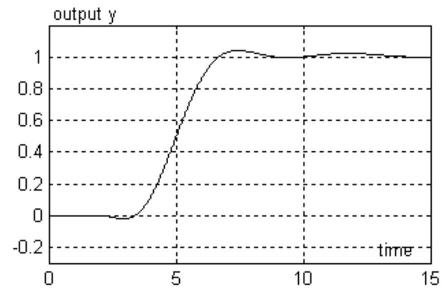

Fig. 6. Step response of the system with Smith predictor and PID regulator

7. Relay control for the plants with delay

In the case of the plants with delay Smith predictor takes the delay outside the loop and owing to this makes it possible to apply some relay control more effectively. One such possibility is applying sliding mode relay control. However this is possible only then, when the replacement plant TF $G_{1S}(s)$ has only minimum phase zeros, because it is the necessary condition. Therefore, if the TF $G(s)$ of the plant contains a delay and some nonmimimum phase zeros, then after applying Smith predictor, the replacement plant TF $G_{1S}(s)$ contains nonmimimum phase zeros and it is impossible to apply the sliding mode control [8].

Another situation is when we apply the parallel compensator, creating the system which may be treated as the system with modified sliding mode control. Consider the system shown in Fig. 7a in which in the place of the P regulator with the gain k it appears the relay with characteristic shown in Fig. 7b. Assume that the hysteresis h of the relay is small and the high frequency oscillations generated by fast switchings of the relay are filtered by the dynamics of the plant $G(s)$, as well as by $G_1(s)$ and $G_c(s)$. Let $\bar{y}(t)$ and $\bar{y}_c(t)$ be the outputs of the plant $G(s)$ and parallel compensator $G_c(s)$, respectively, in which the high frequency oscillations are neglected. Since the amplitudes of these oscillations are small then it is

$$\bar{y}(t) \approx y(t), \quad \bar{y}_c(t) \approx y_c(t). \qquad (24)$$

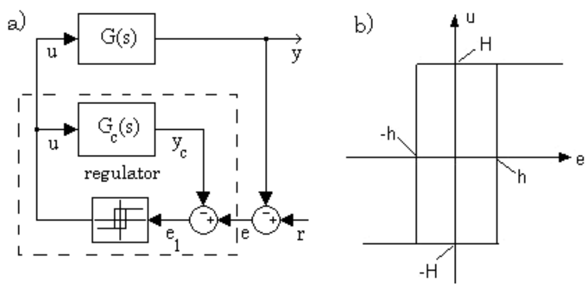

Fig. 7. a) System with parallel compensator and relay; b) characteristic of the relay

During fast switching the relay works on vertical segments of its characteristic, therefore in an approximate description the relay may be treated as the linear static element with very high gain k ($k \to \infty$ when $h \to 0$).

Let $\overline{u}(t)$ be the control signal with filtered high frequency oscillations, containing the slowly varying component such that $\overline{Y}(s) = G(s)\overline{U}(s)$, where $\overline{Y}(s) = \mathcal{L}[\overline{y}(t)]$ and $\overline{U}(s) = \mathcal{L}[\overline{u}(t)]$ where $\mathcal{L}$ denotes Laplace transform. Let $\overline{Y}_c(s) = G_c(s)\overline{U}(s) = \mathcal{L}[\overline{y}_c(t)]$ and $\overline{E}(s) = R(s) - \overline{Y}(s), R(s) = \mathcal{L}[r(t)]$, $\overline{E}_1(s) = \overline{E}(s) - \overline{Y}_c(s)$. During fast switching it is $|e_1| \le h$ and if $h \to 0$ we have $e_1 \approx 0$, $E_1(s) \approx 0$ and $\overline{E}_1(s) \approx 0$. Since $\overline{E}_1(s) = \overline{E}(s) - G_c(s)\overline{U}(s) \approx 0$ then

$$C(s) = \frac{\overline{U}(s)}{\overline{E}(s)} \approx \frac{1}{G_c(s)}. \tag{25}$$

The formula (25) describes the TF of the regulator outlined in Fig. 7a by the dashed line. The TF takes the same form as the formula (8) valid for linear system shown in Fig. 1b. Therefore for the CL system we have

$$\frac{\overline{Y}(s)}{R(s)} = \frac{G(s)/G_c(s)}{1 + G(s)/G_c(s)} = \frac{G(s)}{G_1(s)}. \tag{26}$$

Then the variables $\overline{u}(t), \overline{e}(t), \overline{y}(t)$ and r(t), in which the high frequency oscillations are filtered, are related by the same TF-s as in the linear continuous system from Fig 1a the variables $u(t), e(t), y(t)$ and $r(t)$ are (compare with formulas (8), (9)). Thus the parallel compensator for relay implementation may be designed in the same manner as previously (for continuous, linear implementation).

Strictly speaking the formulas (25), (26) are valid for the system shown in Fig. 7a, if the relay generates fast switching of the control u from $u = -H$ to $u = +H$. In this case the relay may be replaced by the linear amplifier with high gain k. However from the characteristic of the relay it results that the maximal and minimal values of the control u are determined by $+H$ and $-H$, respectively. In the system shown in Fig. 7a the relay sometimes gives the values of the control equal to $+H$ or $-H$, over some period of time without fast switching. One may note that both the cases with and without fast switching may be accounted by replacing the relay in the system shown in Fig. 7a by the amplifier with high gain k (if $h \to 0$ then $k \to \infty$) connected in series with the element with saturations $u_{mx} = H$ and $u_{\min} = -H$, as in Fig. 3b. Both these systems (the first from Fig. 7a with $h \to 0$ and the second from Fig 3b with $k \to \infty$ and $u_{mx} = +H$, $u_{\min} = -H$) give the same time response of the output y to the same reference value r.

7.1. Example 4. Consider a relay control for the plant $G(s)$ (18). Since the plant TF contains the delay and one nonminimum phase zero then applying of Smith predictor is ineffective. Therefore we will apply the parallel compensator. Consider the system with relay shown in Fig. 7a for which the TF-s $G(s)$ and $G_c(s)$ are determined by (18) and (2) together with (19) and (18), respectively. Assume the parameters of the relay as $H = 10$, $h = 0.01$ and the reference value $r = \mathbf{1}(t-1)$ This system will be called here the system with relay.

Consider also the continuous system with saturation shown in Fig. 3a, for which the TF-s $G(s)$, and $G_c(s)$ are the same (determined by (18) and (2) together with (19) and (18), respectively). Assume that the amplifier has the gain $k = 1000$ and the saturation has parameters $u_{mx} = H = 10$, $u_{\min} = -H = -10$. Assume the same reference value $r = \mathbf{1}(t-1)$. This system will be called here the continuous system with saturation.

The time responses of the output y, resulting from simulations are shown in Fig. 8. Solid line is used for the relay system, while dotted line – for the continuous system with saturation. It is seen that in accordance with above considerations the responses of both the systems are almost the same. For smaller h, say $h = 0.005$ they are visually not distinguishable at all. It is seen that smaller the absolute value of saturation (now 10 – in the Example 2 it was 20) gives some insignificant increase of the period of appearance of transients and some deformation of the transient (as it was noted in Example 2). By the way, for the system with relay for which $H = 20$ and $h = 0.01$ the time response of the output y to the reference $r = \mathbf{1}(t-1)$ is almost the same as that shown in Fig. 4a.

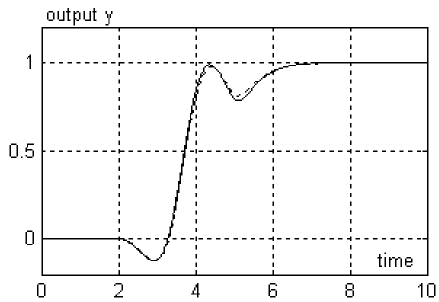

Fig. 8. Step responses: of the relay system – solid line and of the continuous system – dotted line

From these results it is seen, that using the relay control (for which the actuator is usually simpler) we may obtain almost the same quality of control as in the continuous system (in the relay system there appears the chattering effect which sometimes is not accepted by users).

Acknowledgements. The paper contains the results of the research project partially supported by the Ministry of Science and Higher Education in Poland, in 2008.

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