BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES
Vol. 56, No. 1, 2008

Linguistic decomposition technique based on partitioning the knowledge base of the fuzzy inference system

B. WYRWOŁ*

Institute of Electronics, Silesian University of Technology
16 Akademicka St., 44-100 Gliwice, Poland

Abstract. The paper presents Gupta's relational decomposition technique expanded on linguistic level. It allows to reduce the hardware cost of the fuzzy system or the computing time of the final result, especially when referring to First Aggregation Then Inference (FATI) relational systems or First Inference Then Aggregation (FITA) rule systems. The inference result of the hierarchical system using decomposition technique is more fuzzy than of the classical system. The paper describes a linguistic decomposition technique based on partitioning the knowledge base of the fuzzy inference system. It allows to decrease or even totally remove a redundant fuzziness of the inference result.

Key words: membership function, fuzzy rule, fuzzy relation, knowledge base, relational decomposition, linguistic decomposition, First Inference Then Aggregation system (FITA), First Aggregation Then Inference system (FATI), fuzzy inference.

1. Introduction

The general structure of the MISO (*Multiple Inputs Single Output*) FIS (*Fuzzy Logic Inference System*) is shown in Fig. 1. It consists of the following components: a fuzzification block, a knowledge base, an inference block and a defuzzification block [1, 2].

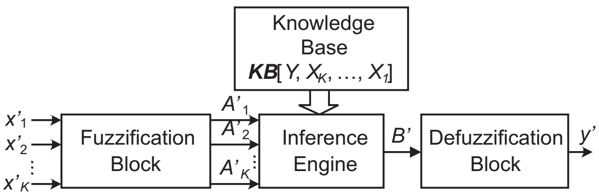

Fig. 1. General structure of the Fuzzy Logic Inference System

The knowledge base $\boldsymbol{KB}[Y, X_K, ..., X_1]$ comprises a collection of linguistic rules and definitions of linguistic variables. The fuzzy system is characterized by the linguistic description in the form of fuzzy rules

$$\begin{aligned} &\text{If } X_K \text{ is } \underline{A}_{Kj_K} \text{ and ... and } X_1 \text{ is } \underline{A}_{1j_1},\\ &\quad \text{then } Y \text{ is } \underline{B}_{j_K \cdots j_2 j_1}, \text{ also...} \end{aligned} \tag{1}$$

where X_K, ..., X_2, X_1 are input variables; Y is an output variable; $\underline{A}_{Kj_K}$, ..., $\underline{A}_{2j_2}$, $\underline{A}_{1j_1}$, $\underline{B}_{j_K ... j_2 j_1}$ are linguistic values defined by fuzzy sets A_{Kj_K}, ..., A_{2j_2}, A_{1j_1}, $B_{j_K ... j_2 j_1}$ on the corresponding universes of discourse X_K, ..., X_2, X_1 and Y respectively ($j_K = 1...N_K$, ..., $j_2 = 1...N_2$, $j_1 = 1...N_1$, where N_k ($k = 1...K$) denotes the number of the linguistic values for the k^{th} input variable).

The general inference process usually encompasses four (or three for a system with exclusively fuzzy outputs) steps:

1) Fuzzification; actual input values x'_K, ..., x'_2, x'_1 are converted into fuzzy sets A'_K, ..., A'_2, A'_1. The most popular method is singleton fuzzification (systems with no fuzzy inputs).
2) Inference; the membership functions defined on the input variables X_K, ..., X_2, X_1 are applied to their actual values A'_K, ..., A'_2, A'_1 to determine the degree of truth for each rule premise (the if-parts of the rules) and then applied to the conclusion part of each rule (the then-parts of the rules).
3) Aggregation; all of the fuzzy subsets obtained in the previous step are combined together to form a single fuzzy set B' for output variable Y (fuzzy output).
4) Defuzzification; converts the fuzzy output set B' to a crisp number y'.

The output fuzzy set $B'_{j_K ... j_2 j_1}$ for rule $R_{j_K ... j_2 j_1}$ (1) can be expressed by means of the formula [1, 2]

$$B'_{j_K ... j_2 j_1} = \boldsymbol{A}' \circ \mathfrak{R}_{j_K ... j_2 j_1} \tag{2}$$

where the symbol $\circ$ denotes the compositional rule of inference operators (e.g. sup-min, sup-prod), $\mathfrak{R}_{j_K ... j_2 j_1}$ represents the relation between antecedent and consequent part of $R_{j_K ... j_2 j_1}$ rule, and $\boldsymbol{A}' = A'_K \times ... \times A'_2 \times A'_1$. The relation $\mathfrak{R}_{j_K ... j_2 j_1}$ defined in the Cartesian product $X_K \times ... \times X_2 \times X_1 \times Y$ can be expressed by the formula (Mamdani) [1–4]

$$\mathfrak{R}_{j_K ... j_2 j_1} = \boldsymbol{A}_{j_K ... j_2 j_1} \wedge B_{j_K ... j_2 j_1}, \tag{3}$$

where $\wedge$ denotes the MIN operator and

$$\boldsymbol{A}_{j_K ... j_2 j_1} = A_{Kj_K} \times ... \times A_{2j_2} \times A_{1j_1}. \tag{4}$$

The single output fuzzy set B' for a collection of rules can be computed on the basis of two approximate reasoning methods:

*e-mail: bernard.wyrwol@polsl.pl

Method 1

The fuzzy sets $B'_{j_K \ldots j_2 j_1}$ are combined together to get a single fuzzy set by using the aggregate MAX ($\vee$) operator

$$B' = \bigvee_{j_K=1}^{N_K} \ldots \bigvee_{j_1=1}^{N_1} B'_{j_K \ldots j_2 j_1} \tag{5}$$

Method 2

A global relation $\Re$ for all rules is determined

$$\Re = \bigvee_{j_K=1}^{N_K} \ldots \bigvee_{j_1=1}^{N_1} \Re_{j_K \ldots j_2 j_1} \tag{6}$$

and then the output fuzzy set is computed according to the formula

$$B' = \boldsymbol{A}' \circ \Re, \tag{7}$$

where the symbol $\circ$ denotes the compositional rule of inference operators (e.g. sup-min, sup-prod).

2. Gupta's decomposition method

A decomposition technique based on projection of the global fuzzy relation $\Re$ first proposed by M.M. Gupta, B. Kiszka and G.M. Trojan is presented in [4]. It makes it possible to convert the global multidimensional relation $\Re$ into a set of two-dimensional subrelations $\Re_i$ ($i = 1, \ldots, K$), thus the classical relational FATI system can be implemented as a hierarchical architecture that comprises a set of SISO (*Single Input Single Output*) relation-based subsystems. The decomposition technique reduces hardware cost of the fuzzy system, but computation of the global relation $\Re$ is extremely time-consuming process and a large memory is necessary to store the relation (hardware cost and computational time depends on number of inputs and outputs of the system i.e. the dimensionality of the fuzzy relation [6]). The global fuzzy relation $\Re$ is used only to perform decomposition process and it is no longer necessary afterwards. These disadvantages can be eliminated if decomposition is used for knowledge base ***KB***$[Y, X_K, \ldots, X_1]$ of the FITA system (the fuzzy relation $\Re$ of the FATI inference system is computed based on information stored in knowledge base ***KB***$[Y, X_K, \ldots, X_1]$ (where $Y, X_K, \ldots, X_1$ stand for outputs and inputs linguistic variables, respectively) of the FITA inference system [1, 3]. The proposed methodology assumes transformation of a classical projection of fuzzy relation [4] into linguistic projection of knowledge base [13]. It can be expressed by the formula

$$\boldsymbol{KB}_i[Y, X_1] = \underset{X_K, \ldots, X_{i+1}, X_{i-1}, \ldots, X_1}{proj} \boldsymbol{KB}[Y, X_K, \ldots, X_1] \tag{8}$$

where *proj* is a projection operation on linguistic level [13]. It creates the knowledge bases $\boldsymbol{KB}_i[Y, X_i]$ (i=1, …, K) through elimination of all the input linguistic variables except for X_i in the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$. Antecedent parts of fuzzy rules remain unaltered (linguistic values defined by fuzzy sets $A_{Kj_K}, \ldots, A_{2j_2}, A_{1j_1}$), but linguistic values of the consequence parts of fuzzy rules in the knowledge bases $\boldsymbol{KB}_i[Y, X_i]$ (linguistic values described by fuzzy sets $B^D_{ij_i}$) are computed by combination of the consequence parts of fuzzy rules in the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$ (described by fuzzy sets $B_{j_K \ldots j_2 j_1}$) according to formula

$$B^D_{ij_i} = \bigvee_{j_K=1}^{N_K} \ldots \bigvee_{j_{i+1}=1}^{N_{i+1}} \bigvee_{j_{i-1}=1}^{N_{i-1}} \ldots \bigvee_{j_1=1}^{N_1} B_{j_K \ldots j_2 j_1} \tag{9}$$

where $N_K, \ldots, N_1$ denote number of the linguistic values of the input linguistic variable $X_K, \ldots, X_1$, while $\vee$ denotes the MAX operator [3, 4].

Figure 2 presents linguistic projection of a three-dimensional primary knowledge base (dimension of a knowledge base is equal to the number of its linguistic variables [5, 6]).

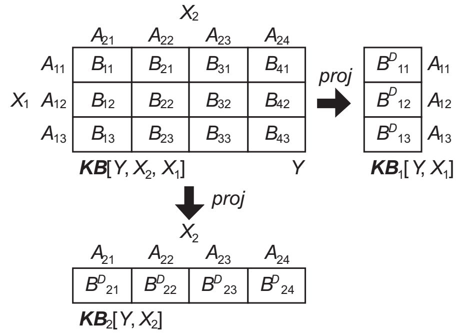

Fig. 2. Graphical illustration of the linguistic projection of a three-dimensional knowledge base into two two-dimensional knowledge bases

For example, the fuzzy sets B^D_{12} and B^D_{23} are computed according to the formulas

$$\begin{aligned} B^D_{12} &= B_{12} \vee B_{22} \vee B_{32} \vee B_{42}, \\ B^D_{23} &= B_{31} \vee B_{32} \vee B_{33}. \end{aligned} \tag{10}$$

The obtained knowledge bases $\boldsymbol{KB}_i[Y, X_i]$ (8) can be used for implementation of the FITA hierarchical system or a one that is converted into subrelation $\Re_i$ and used to implementation of the FATI system (Fig. 3). In this case, computation of a global fuzzy relation $\Re$ is not required. The FIS and FIS$_i$ (i=1, …, K) modules are FITA/FATI systems and have logical architectures presented in [7].

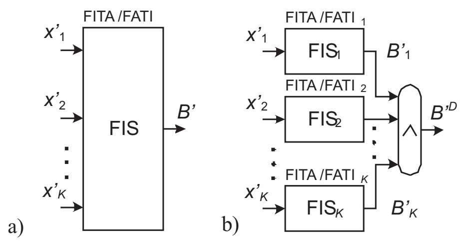

Fig. 3. Logical architectures of the inference system: (a) classical, (b) hierarchical

The Gupta's decomposition technique allows a decrease in the hardware cost of the fuzzy system and additionally, expanded on linguistic level, makes it possible to avoid calculation the global fuzzy relation $\Re$ (it reduces computation time and storage requirements), enables implementation of the FITA system and analyzing behaviour of the fuzzy subsystems in the hierarchical architecture and the entire system in a simple way.

However, the method cannot be applied to every case, due to the fact that the inference result in classical and hierarchical architectures of the inference system may differ. In some cases behaviour of the systems will be different and the decomposition in this form can not be used.

3. Partitioning the knowledge base of the inference system

The projection operation (on relational or linguistic level) in some cases can lead to inevitable loss of an information because of its approximate nature [6]. Thus the output inference result in the hierarchical system is deformed. It is not possible to reconstruct the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$ from the subbases $\boldsymbol{KB}_i[Y, X_i]$ obtained by the linguistic decomposition method. It can be noticed that in the reconstructed knowledge base $\boldsymbol{KB}^{ce}[Y, X_K, \ldots, X_1]$ some of the primary rules have been deformed: instead of the rule appointed as $R_{J_K \ldots J_2 J_1}$ ($J_K = 1, \ldots, N_K; \ldots; J_1 = 1, \ldots, N_1$) the rule $R^{ce}_{J_K \ldots J_2 J_1}$ has appeared. The linguistic value of the consequence part of the $R^{ce}_{J_K \ldots J_2 J_1}$ rule can be described by the fuzzy set:

$$B^{ce}_{J_K \ldots J_1} = B_{J_K \ldots J_1} \vee \left(\bigvee_{j_K=1}^{N_K} \ldots \bigvee_{j_1=1}^{N_1} (B_{J_K \ldots j_2 j_1} \wedge \ldots \\ \ldots \wedge B_{j_K \ldots J_2 j_1} \wedge B_{j_K \ldots j_2 J_1}) \right) \tag{11}$$

where N_k ($k = 1 \ldots K$) denotes the number of the linguistic values for the k^{th} input variable X_k, while $\vee$ and $\wedge$ stand for the MAX and MIN operators, respectively. If the product of the fuzzy sets $B_{J_K \ldots j_2 j_1} \wedge \ldots \wedge B_{j_K \ldots J_2 j_1} \wedge B_{j_K \ldots j_2 J_1}$ ($j_K = 1, \ldots, N_K; \ldots; j_1 = 1, \ldots, N_1$) is not an empty set then an inference error appears. All the rules $R_{j_K \ldots j_2 j_1}$ (for the specific rule $R_{J_K \ldots J_1}$), for which the aforementioned product is not an empty set, are referred to as the inconsistent rules [7].

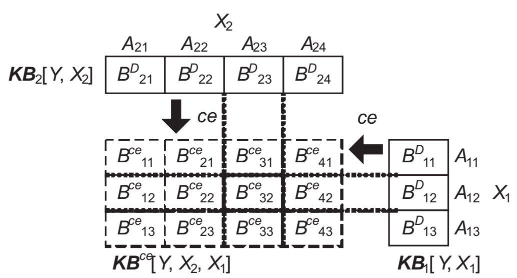

Fig. 4. Graphical illustration of the reconstructing process for the primary knowledge base from its subbases by means of the operation of cylindrical extension on linguistic level

Figure 4 shows an example of the reconstructed knowledge base $\boldsymbol{KB}^{ce}[Y, X_2, X_1]$ that can be created from two subbases $\boldsymbol{KB}_1[Y, X_1]$ and $\boldsymbol{KB}_2[Y, X_2]$ by means of the cylindrical extension operation expanded on the linguistic level [7].

The linguistic value of the consequence part of the rule R^{ce}_{32} ($J_2 = 3$, $J_1 = 2$) that is highlighted in Fig. 4 can be described by the fuzzy set:

$$B^{ce}_{32} = B^D_{12} \wedge B^D_{23}, \tag{12}$$

where fuzzy sets B^D_{12} and B^D_{23} are defined by formula (10). Let assume that exclusively for fuzzy sets B_{12} and B_{31} their product is not an empty set ($B_{12} \wedge B_{31} \neq \varnothing$) (primary knowledge base in Fig. 2), then Eq. (12) can be expressed as

$$B^{ce}_{32} = B_{32} \vee (B_{12} \wedge B_{31}). \tag{13}$$

The rules R_{12} ($j_2 = 1$, $J_1 = 2$) and R_{31} ($J_2 = 3$, $j_1 = 1$) are inconsistent for the specific rule R_{32} ($J_2 = 3$, $J_1 = 2$) under consideration.

To avoid deformation of the output inference result, a modified Gupta's decomposition technique has been proposed, which is applicable to all cases. The method is based on removing the inconsistent rules from the primary knowledge base. It can change behaviour of some systems. Thus, the primary knowledge base should be expressed as a sum of p knowledge bases

$$\boldsymbol{KB}[Y, X_K, ..., X_1] = \bigvee_p \boldsymbol{KB}_p[Y, X_K, ..., X_1] \tag{14}$$

where each of knowledge base $\boldsymbol{KB}_p[Y, X_K, \ldots, X_1]$ includes all or only some selected rules $R_{j_K \ldots j_2 j_1}$ ($j_K = 1, \ldots, N_K; \ldots, j_1 = 1, \ldots, N_1$) extracted from the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$. It is important that each rule from the primary knowledge base appears in knowledge subbases $\boldsymbol{KB}_p[Y, X_K, \ldots, X_1]$ at least once and neither of the knowledge subbases includes inconsistent rules. The operation described above is referred to as partitioning the knowledge base of fuzzy inference systems. It is partitioning the primary knowledge base into p knowledge subbases. They have the same size, but various contents. The linguistic Gupta's decomposition can be used for each subbase without loss of information, thus an inference result in classical and decomposed system is the same.

The most important problem in the method under consideration is how to achieve optimum partitioning of the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$. Number of newly created subbases (parameter p in Eq. (14)) should be as small as possible and each of them should be free of inconsistent rules. The second provision is achieved if

$$\bigvee_{j_K=1}^{N_K} \ldots \bigvee_{j_1=1}^{N_1} (B_{J_K \ldots j_2 j_1} \wedge \ldots \wedge B_{j_K \ldots j_2 J_1}) = \begin{cases} B_{J_K \ldots J_1} \\ \text{or} \\ \varnothing \end{cases} \tag{15}$$

where $\varnothing$ denotes an empty fuzzy set.

The partitioning of the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$ can be performed in many various ways. The easiest and the fastest method is based on partitioning towards a defined input linguistic variable X_r (the PDILV method [7]). In this case the number of newly created subbases $\boldsymbol{KB}_{J_K \ldots J_{r+1}, J_{r-1} \ldots J_1}[Y, X_K, \ldots, X_1]$ is given by the formula

$$p = N_K \cdot \ldots \cdot N_{r+1} \cdot \ldots \cdot N_1 \tag{16}$$

The knowledge subbase $\boldsymbol{KB}_{J_K \ldots J_{r+1}, J_{r-1} \ldots J_1}[Y, X_K, \ldots, X_1]$ comprises only the rules $R_{J_K \ldots j_r \ldots J_2 J_1}$ ($j_r = 1, \ldots, N_r$) that are descended from the primary knowledge base. The selected input variable is that one, for which the number of linguistic values is the largest. In this case the p coefficient (Eq. (16)) is the smallest and the sum of products of fuzzy sets in Eq. (15) is an empty set. Graphical illustration of the knowledge base partitioning for the fuzzy inference system into the knowledge base without inconsistent rules is shown in Fig. 5.

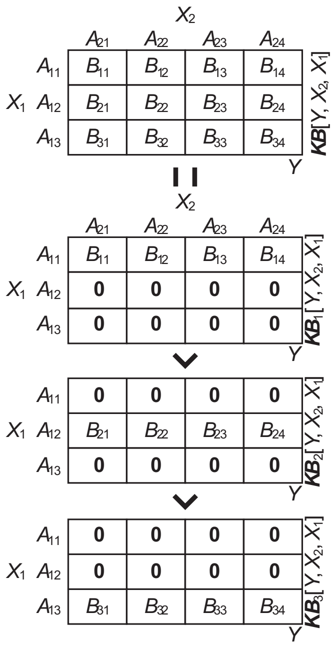

Fig. 5. Graphical illustration of the partitioning the knowledge base of the fuzzy inference system into the knowledge base without inconsistent rules

The example knowledge base is partitioned into three subbases towards the X_2 linguistic variable (number of linguistic value for it is 4). Number of linguistic values for variable X_1 is three, thus number of subsystems would be four. The last result is not acceptable, because the first result is lower.

4. Architecture of the inference system

The logical architecture of the fuzzy inference system for the proposed decomposition method is shown in Fig. 6.

It consists of the p subsystems HFIS$_q$ ($q = 1, \ldots, p$), each of them is composed of K SISO (*Single Input Single Output*) systems (in Fig. 5 marked as FIS$_{pk}$; $k = 1, \ldots, K$; p depends on the decomposition technique: for primary Gupta's decomposition technique $p = 1$). They can be implemented as a rule (FITA) or a relational (FATI) fuzzy inference engines [7, 8]. The partial fuzzy inference results (in a form of fuzzy sets $B_q'^D$) are added as an additional component. This component produces a sum with use of the MAX operator [1, 2, 4]. The behaviour of each subsystem depends on the content of its knowledge base $\boldsymbol{KB}_p[Y, X_K, \ldots, X_1]$ that has been created as a result of partitioning the primary knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$. The HFIS$_q$ ($q = 1, \ldots, p$) subsystem consists of FIS$_{qk}$ ($q = 1, \ldots, p$; $k = 1, \ldots, K$) SISO inference systems. They are created as a result of performing the primary Gupta's decomposition technique on linguistic level for the knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$. As a result the knowledge bases $\boldsymbol{KB}_{qk}[Y, X_k]$ ($k = 1, \ldots, K$) have been obtained. They describe behaviour of each SISO system FIS$_{qk}$ in the hierarchical structure. The basic SISO systems FIS$_{qk}$ can be implemented as FITA (in this case the information from the knowledge bases $\boldsymbol{KB}_{qk}[Y, X_k]$ has been used during the inference process) or FATI (in this case the information from the knowledge bases $\boldsymbol{KB}_{qk}[Y, X_k]$ has been converted to the form of fuzzy relation $\mathfrak{R}_{qk}$) fuzzy inference engines.

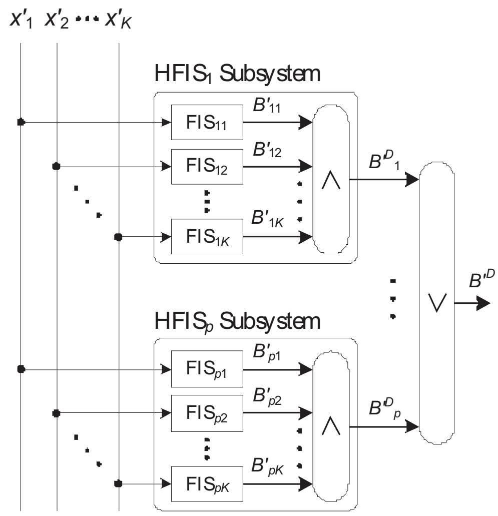

Fig. 6. Logical architecture of the fuzzy inference system without defuzzification module of the inference result

5. Summary

The number of subsystems HFIS in any hierarchical fuzzy inference system for primary Gupta's decomposition method is equal $p = 1$. To avoid inference error (the output result is more fuzzy than the output result obtained in the classical system architecture) when a modified decomposition technique based on partitioning the knowledge base $\boldsymbol{KB}[Y, X_K, \ldots, X_1]$ is applied, the number of subsystems p in general case is greater than 1. The number of subsystems should be increased to avoid inference error, and consequently the hardware cost of the system may increase as well (this problem is not crucial for most FATI systems). The number of newly created subsystems HFIS should be as low as possible. In Section 3, the method based on partitioning the knowledge base is proposed. The algorithm is simple and fast, but the results are not optimal in all cases. Table 1 brings together amounts of HFIS subsystems for several fuzzy inference systems (used as benchmarks). The knowledge bases of the fuzzy inference systems, mentioned in Table 1, describe respectively: fuzzy controllers (denominated as 1, 3, 4) [4, 9, 10], ENOR gate (2) [6], truck park controller (5) [2], temperature controller of a heated air-stream (6) [11], fuzzy controller for stabilization of an inverted pendulum (7) [12], fan controller (8) [13] and a fuzzy system for identification of nonlinear systems (9) [14]. It can be noticed that only for four fuzzy systems results are optimal.

The estimated hardware cost of the fuzzy inference system can be expressed as

$$H \approx H_{P_{mem}} + H_{L_{conn}} + H_{L_{comp}}, \tag{17}$$

where $H_{P_{mem}}$, $H_{L_{conn}}$ and $H_{L_{comp}}$ denote the hardware cost of the memory modules, connections and components used in the system, respectively [7]. The hardware cost can be calculated for classical (H) and hierarchical (H^D) structures of the models described in Section 2. To compare the two structures a hardware cost reduction coefficient has been defined

$$\upsilon_H[\%] = \frac{H - H^D}{H} \cdot 100. \tag{18}$$

Table 2 brings together results for several fuzzy inference systems mentioned above (benchmarks). Attention should be drawn to the fact that only for relational fuzzy inference systems the hardware cost reduction coefficient is very high. For rule fuzzy inference systems (FITA), the decomposition method does not lead to lowering of hardware cost. If the modified decomposition method based on partitioning knowledge base (PDILV) has been used, the final results are even worse (results are optimal for merely two systems). It is important to find an optimal value of p parameter, but it is not guaranteed, in the aspect of FITA systems, that the hardware cost coefficient greater than zero would be achieved. If the fuzzy inference system is implemented as a rule-relational one the benefits (the FATI system) exceeded wastages, and the results, achievable with use of the proposed method, are acceptable.

Table 1
Number of HFIS subsystems for some inference systems

Parameter	Realization method	Inference system (benchmark)								
		1	2	3	4	5	6	7	8	9
Number of HFIS subsystems (p)	Optimal value	3	2	5	3	5	2	3	2	3
	PDILV	5	2	5	3	5	3	5	3	5

Table 2
Hardware cost reduction using the partitioning knowledge base towards definite input linguistic variable method (PDILV)

Parameter	Realization method (decomposition)	Inference system (benchmark)								
		1	2	3	4	5	6	7	8	9
Relational Fuzzy Inference System (FATI)										
Hardware cost reduction [%]	Classic Gupta's	98	98	98	98	98	98	98	98	98
	PDILV	91	94	86	91	86	91	91	94	91
Rule Fuzzy Inference System (FITA)										
Hardware cost reduction [%]	Classic Gupta's	55	−4	51	23	47	34	22	23	49
	PDILV	16	−38	−27	−38	−21	−25	−9	−6	16

REFERENCES

[1] E. Czogała and W. Pedrycz, *Elements and Methods of Fuzzy Set Theory*, PWN, Polish Scientific Publishers, Warsaw, 1985.

[2] D. Rutkowska, M. Pilinski, and L. Rutkowski, *Neural Networks, Genetic Algorithms and Fuzzy Systems*, PWN, Polish Scientific Publishers, Warsaw, 1997.

[3] D. Driankov, H. Hellendoorn, and M. Reinfrank, "An introduction to Fuzzy Control", WNT, Warszawa, 1996.

[4] R.R. Yager and D.P. Filev, *Principles of Modelling and Fuzzy Control*, WNT, Warszawa, 1995, (in Polish).

[5] M.M. Gupta, J.B. Kiszka, and G.M. Trojan, "Multivariable structure of fuzzy control systems", *IEEE Transactions on Systems, Man, and Cybernetics* 16 (5), 638–656 (1986).

[6] P.G. Lee, K. Lee Kyun, and G.J. Jeon, "An index of applicability for the decomposition method of multivariable fuzzy systems", *IEEE Transactions on Fuzzy Systems* 3 (3), 364–369 (1995).

[7] B. Wyrwoł, *Hardware Realisation of Approximate Inference with the Use of Programmable Logic Systems*, PhD Thesis, Silesian University of Technology, Gliwice, 2004.

[8] B. Wyrwoł, "The rule-relation system of approximate inference", *IV State Conf. on Electronics* 2, 475–480 (2005).

[9] I. Baturone, S. Sanchez-Solano, A. Barriga, and J.L. Huertas, "Implementation of CMOS fuzzy controllers as mixed-signal integrated circuits", *IEEE Transactions on Fuzzy Systems* 5 (1), 1–19 (1997).

[10] D. Kim and In–Hyun Cho, "An accurate and cost–effective COG defuzzifier without the multiplier and the divider", *Fuzzy Sets and Systems* 104, 229–244 (1999).

[11] A. Ollero and A.J. Garcia-Cerezo, "Direct digital control, auto–tuning and supervision using fuzzy logic", *Fuzzy Sets and Systems* 30, 135–153 (1989).

[12] T. Yamakawa, "Stabilization of an inverted pendulum by a high-speed fuzzy logic controller hardware system", *Fuzzy Sets and Systems* 32, 161–180 (1989).

[13] H.D. Hurdon, *Fuzzy Logic Fan Controller*, ntia.its.bldrdoc.gov/pub/fuzzy (1993).

[14] R. Rovatti, R. Guerrieri, and G. Baccarani, "An enhanced two-level Boolean synthesis methodology for fuzzy rules minimization", *IEEE Trans. on Fuzzy Systems* 3, 288–299 (1995).