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The recoil nature of electrostatic and gravitational forces

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Abstract. The paper explains the force induced by the electrostatic field on the electron as a recoil force. The starting point is the hypothesis that in the dynamic equilibrium with the vacuum, the electron simultaneously absorbs and emitts energy. With no external electrostatic field the radiation patterns of absorption and emission are assumed to be isotropic. The external electrostatic field induces anisotropy of the emission resulting in a recoil force. The paper presents a theoretical description of this force using a model of the angular power density pattern of the emission in the form of an ellipsoid. Calculations show that the total radiated power is extremely high. This radiation is compared with the electromagnetic radiation of the electron on the Bohr orbit in the idealized hydrogen atom. An analogous problem for gravitational forces is presented.

1. Introduction

This paper is a modified version of a similar paper written by the author in 1976 [1]. We decided to write the modification for two reasons: Firstly, the paper [1] was written in German and printed in Kleinheubacher Berichte which has a limited number of readers. Secondly, in recent years we observed a better understanding of the nature of quantum vacuum as a medium of extremely high energy density. Note, that the paper [1] was based on ideas concerning the nature of the vacuum presented by Vallée in [2].

2. Preleminaries

For convenience, the definitions and notations of all notions used in this paper are presented in the Appendix 2 in Table 1. Consider a uniform electrostatic field $\vec{E}_x = E_0\bar{x}$, where $\bar{x}$ is a versor directed along the x – axis in the 3-D space $\Re$. Let us assume that a single electron can be represented by a point charge e. The external electrostatic field $\vec{E}_x$ induces a force

$$\vec{F}_x = e\vec{E}_x. \tag{1}$$

Let us assume that this equation holds for finite dimensions of the electron. An analogous formula for a point mass m and the gravitational field $\vec{G}_x = G_0\bar{x}$ is

$$\vec{F}_x = m\vec{G}_x. \tag{2}$$

The energy density of the electrostatic field is

$$E_E = 0.5\varepsilon_0 E_0^2 \tag{3}$$

where ε_0 is the permittivity of the free space. The analogous energy density of the gravitational field is

$$E_G = -0.5\gamma G_0^2 \tag{4}$$

where γ is the gravitational permittivity of the free space (see Table 1). Note the minus sign which indicates that the energy level of the gravitational field is lower than the energy of the zero-field vacuum. This fact has been described in [2]. A simple evidence is given is Appendix 2.

3. Energy density of the quantum vacuum

The energy density of the quantum vacuum can be derived using the Planck's formula

$$\rho\left(f,T\right) = \frac{8\pi f^2}{c^3}\left[\frac{hf}{e^{\frac{hf}{kT}}-1} + \frac{hf}{2}\right] \tag{5}$$

where h is the Planck constant, k – the Boltzmann constant, T – the absolute temperature and f – the frequency of the radiation. For $T = 0$ we get

$$\rho\left(f,T=0)\right) = \frac{4\pi h f^3}{c^3}. \tag{6}$$

The term $hf/2$ represents the zero-point fluctuations of the quantum vacuum. The quantum vacuum is not empty (void) but is a medium of extremely high energy density. The total energy density in the frequency band from f_1 to f_2 is

$$\rho = \int_{f_1}^{f_2} \rho\left(f\right) df = \frac{\pi h}{c^3}\left(f_2^4 - f_1^4\right). \tag{7}$$

Examples.

1. Planck suggested that the highest frequency of the quantum vacuum is defined by the formula

$$f_{\max} = f_2 = \sqrt{\frac{c^5}{2hG}} \simeq 5.235 \times 10^{42}\ \text{[Hz]}. \tag{8}$$

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Table 1
Notations, definitions, values and dimensions (in SI) of selected physical quantities

Notation	Name	Definition	Value	SI Units
μ_0	Permeability of free space	Defined value	$4\pi\times 10^{-7}$	Hy/m
c	Speed of light in free space	$c = 1/\sqrt{\varepsilon_0\mu_0}$	2.99792458×10^{8}	M/s
ε_0	Permittivity of free space	$\varepsilon_0 = 1/\left(\mu_0 c^2\right)$	$8.85418781\times 10^{-12}$	F/m
h	Planck's constant		6.626176×10^{-34}	Js
$\hbar$	Modified Planck's constant	$\hbar = h/2\pi$		Js
e	Elementary charge		1.6021892×10^{-19}	C
m_e	Rest mass of the electron		9.109514×10^{-31}	kg
m_p	Rest mass of the proton		$1.67262171\times 10^{-27}$	kg
Z_0	Impedance of free space	$Z_0 = \sqrt{\mu_0/\varepsilon_0}$	376.7303	Ω
R_{H}	Quantized Hall resistance	$R_{\mathrm{H}} = h/e^2$	25812.81	Ω
r_B	Bohr radius	$r_B = \left[\varepsilon_0/\left(\pi m_e\right)\right](h/e)^2$	5.2917721×10^{-11}	m
α	Fine structure constant	$\alpha = Z_0/(2R_{\mathrm{H}})$	7.2973506×10^{-3}	
α^{-1}	Reciprocal of α		137.03604	
λ_e	Compton wavelength of the electron	$\lambda_e = \frac{h}{m_e c}$	2.4263089×10^{-12}	m
f_e	Compton frequency of the electron	$f_e = \frac{c}{\lambda_e} = \frac{m_e c^2}{h}$	1.235590×10^{20}	Hz
f_p	Compton frequency of the proton	$f_p = \frac{c}{\lambda_p} = \frac{m_p c^2}{h}$	2.268676×10^{23}	Hz
$r_{class.}$	Classic radius of the electron	$r_{class.} = \frac{\mu_0 e^2}{4\pi m_e}$	2.8179383×10^{-15}	m
r_e	Compton radius of the electron	$r_e = \frac{\lambda_e}{4\pi} = \frac{h}{4\pi m_e c}$	1.9300795×10^{-13}	m
λ_p	Compton wavelength of the proton	$\lambda_p = \frac{h}{m_p c}$	1.3214099×10^{-15}	m
$E_{\max}$	The maximum possible value of the electric	$\lvert E_{\max}\rvert = \frac{e}{4\pi\varepsilon_0 r_e^2}$	3.8625×10^{16}	V/m
	fields defined by the cylindrical model of the electron	$= 4\pi\left(m_e c^2\right)^2\left(e/h^2\right)$		
μ_B	Bohr magneton	$\mu_B = eh/(4\pi m_e c)$	9.274078×10^{-24}	Am^2 or JT^{-1}
λ	Gravitational permittivity of free space	$\gamma = \frac{1}{4\pi G}$	1.1927×10^{9}	kg s^2/m^3
G	Gravitational constant	Measured	6.6720×10^{-11}	m^3/kgs^2
η	Gravitational permeability of free space	$\eta = \frac{16\pi G}{c^2}$	3.7315×10^{-26}	m/kg

This value of f_2 with $f_1 = 0$ yields

$$\rho_{\max} = \frac{\pi h f_{\max}^4}{c^3} \simeq 5.8\times 10^{112}\ [\text{J/m}^3]. \tag{9}$$

We may assume, that this total energy density of the quantum vacuum is infinite.

2. Consider the frequency band of visible light with $f_1 = 0.43\times 10^{15}$ [Hz] and $f_2 = 0.75\times 10^{15}$ [Hz]. We get $\rho = 22$ [J/m^3]

In the next two examples we calculate the value of the frequency band located around the Compton frequency of the particle which yields the energy density of the quantum vacuum equal to the energy density defined by the division of the Einstein's energy $E = mc^2$ by the volume of the particle.

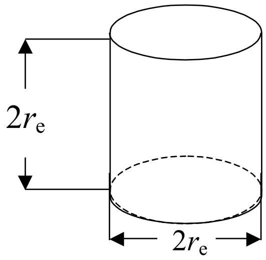

Fig. 1. The cylindrical model of the electron of reference after Ref. 2

3. The volume of the cylinder model of the electron is (see Fig. 1) is

$$V_e = 2\pi r_e^3 = 2\pi\frac{h^3}{(4\pi m_e c)^3} = 2.0750\times 10^{-39}\ [\text{m}^3] \tag{10}$$

This yields the energy density defined by the equation

$$\rho_e = \frac{m_e c^2}{V_e} = \frac{32\pi^2 m_e^4 c^5}{h^3} = 1.81022\times 10^{24}\ [\text{J/m}^3]. \tag{11}$$

In this example we insert in Eq.(7) $f_2 = \mathrm{a} f_e$ and $f_1 = 0$ getting the energy density (f_e – see Table 1)

$$E_e = \frac{\pi h}{c^3} a^4 f_e^4 = \frac{\pi a^4 m_e^4 c^5}{h^3}. \tag{12}$$

Equations (11) and (12) yield $a^4 = 4\pi$, *i.e.*, $a \approx 3.166$. Concluding, the Einstein's energy of the electron corresponds to the energy of the vacuum in a wide frequency band extending from zero to about $3.166\times$the Compton frequency of the electron.

4. Let us present a similar calculation for the proton. We start with a measured value of the radius of the proton $r_p = 0.8\times 10^{-15}$[m] and the mass $m_p = 1.6726485\times 10^{-27}$[kg]. Assuming a spherical model of the proton, the corresponding volume is $V_p = (4/3)\pi r_p^3 = 2.1441\times 10^{-45}$ [m^3]. This yields the energy density

$$\rho_p = \frac{m_p c^2}{V_p} = 7.01166\times 10^{34}\ [\text{J/m}^3] \tag{13}$$

In this example we insert in (7) $f_2 = f_p(1 + a)$ and $f_1 = f_p(1 - a)$, where the Compton frequency of the proton is

$$f_p = \frac{m_p c^2}{h} = \frac{c}{\lambda_p} = 2.268687 \times 10^{23}. \tag{14}$$

We have $f_2^4 - f_1^4 = f_p^4\left[(1+a)^4 - (1-a)^4\right] = 8f_p^4\left(a + a^3\right)$. The insertion in (7) and equating with (13) yields

$$\frac{8\pi h}{c^3}\left[a + a^3\right] = 7.01166 \times 10^{34}. \tag{15}$$

We get $(a + a^3) = 0.043$ giving $a \approx 0.0429$. We observe that differently to the case of electron, the frequency band around the Compton frequency of the proton yielding the Einstein's energy density is narrow and equals to $\Delta f = 0.0858 f_p$.

3.1. The model of the electron used in [2] to calculate the highest possible value of the electric field strength. The author of [2] proposed a cylindrical model of the electron shown in Fig. 1. The radius of the base equals to

$$r_e = \frac{\lambda_e}{4\pi} = \frac{h}{4\pi m_e c} = 1.938 \times 10^{-13}\ [\text{m}], \tag{16}$$

where λ_e is the Compton wave-length of the electron. The height of the cylinder equals $2r_e$. The author of [2] assumed, that the elementary charge is uniformly distributed on the surface $S = 4\pi r_e^2$ of the side wall of the cylinder. We have the relation

$$e = S\varepsilon_0 E_{\max} \tag{17}$$

where $E_{\max}$ is the field strength at the border of the side-wall of the cylinder. We get

$$E_{\max} = \frac{e}{\varepsilon_0 S} = \frac{e}{4\pi\varepsilon_0 r_e^2} = 4\pi\mu_0 f_e^2 e = 3.8625 \times 10^{16}[\text{V/m}]. \tag{18}$$

If we apply instead of the cylinder a sphere of radius r_s and assume a uniform distribution of the charge on the surface of this sphere, then

$$E_{\max} = \frac{e}{4\pi\varepsilon_0 r_s^2} \tag{19}$$

and we get $r_s = r_e$, i.e., the same as the radius of the cylinder. Since the energy density of the electrostatic field as a function of the radius r of a sphere is $\rho_E = 0.5\varepsilon_0 E^2(r)$, the total energy outside the sphere is

$$\int_{r_0}^{\infty} 4\pi r^2 \rho(r)\, dr = \frac{e^2}{8\pi\varepsilon_0 r_0}. \tag{20}$$

Let us compare this energy with $km_e c^2$ $(0 < k < 1)$. We get

$$r_0 = \frac{1}{k}\frac{\mu_0 e^2}{8\pi m_e}. \tag{21}$$

For $k = 0.5$ we get $r_0 = r_c$, where r_c is the classical radius of the electron. For a spherical model of the electron with classic radius, the energy of the external field equals one half of $m_e c^2$. It can be shown, that $r_e = \frac{1}{2}\alpha^{-1} r_c$ (α – fine structure constant), i.e., the radius of the cylindrical model proposed in [2] is about $\alpha^{-1}/2 \approx 68.5$ times longer than the classical radius. As well, the electric field strength at the border of the sphere of classical radius would be $\alpha^{-2}/4$ greater with respect to $E_{\max}$ defined by (18). The insertion in (21) of $r_0 = r_e$ yields

$$k = k_e = \alpha = 7.727... \times 10^{-3}. \tag{22}$$

This calculation holds for the spherical model and with a good approximation for the cylindrical model. For these models, the energy of the external electrostatic field is negligible in comparison to the Einstein's rest energy of the electron. However, (16) is derived assuming, that the energy inside the cylindrical model exactly equals $m_e c^2$.

4. The hypothesis about the recoil nature of electrostatic forces

Let us present derivations describing the electrostatic force (1) as a recoil force. The derivations are based on the hypothesis that the electron is in a dynamic equilibrium with the energetic medium of the quantum vacuum. It is assumed that the electron continuously absorbs and emits radiation. In absence of any external electrostatic field, it is assumed that the directional patterns of absorption and emission are isotropic (for the spherical model) or circularly symmetric (for the cylindrical model). It is assumed that the external electrostatic field induces anisotropy of the emission while the absorption remains isotropic. In that case. the anisotropy of the emission induces a recoil force given by the integral

$$\left|\vec{F}\right| = \left|\frac{|\vec{v}|}{c^2}\int_{4\pi}\sigma_\Omega \cdot \vec{n}_0 d\Omega\right|_{\max} \tag{23}$$

where σ_Ω [W/Ster] is the angular power density of the radiation (power per unit solid angle), $|\vec{v}|$ the velocity of the radiation and $\vec{n}_0$ – a versor, which yields the maximum value of the integral.

Let us investigate the case for the power density diagram given by the ellipsoid (see Fig.2)

$$\sigma_\Omega = \sigma_{\max}\frac{1-\varepsilon^2}{1+\varepsilon\cos(\varphi)} \tag{24}$$

where ε is the eccentricity of the ellipsoid. If $\varepsilon \ll 1$, the derivation presented in the Appendix 1 yields

$$\left|\vec{F}\right| = \frac{|\vec{v}|}{3c^2}P\varepsilon, \tag{25}$$

where P is the total radiated power The force (25) should be equal to the electrostatic force $e\left|\vec{E}\right|$. This yields the following expression for the power P

$$e\left|\vec{E}\right| = \frac{\left|\vec{v}\right| P\varepsilon}{3c^2} \;\rightarrow\; P = \frac{3e\left|\vec{E}\right| c^2}{\left|\vec{v}\right| \varepsilon}. \qquad (26)$$

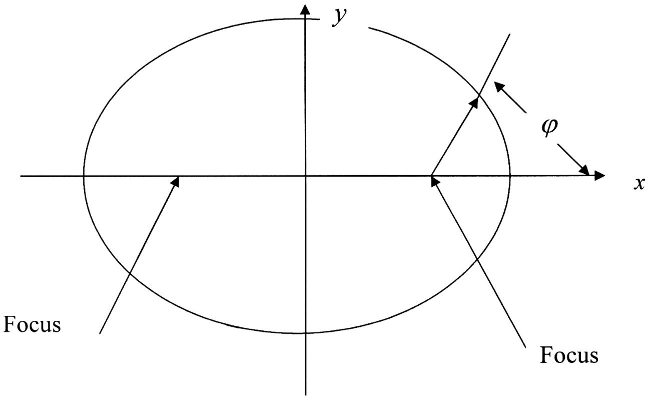

Fig. 2. The rotation of the ellipse around the major x – axis defines the ellipsoid

In the derivation in the Appendix 1 we suggested that the eccentricity $\varepsilon \approx \frac{|\vec{E}|}{|\vec{E}_{\max}|}$. Therefore, assuming $[\vec{v}) = c$, we get

$$P = 3e\left|\vec{E}_{\max}\right| c = 12\pi e^2 f_e^2 Z_0 = 2, 2.7859\times 10^6 \text{ [W]}. \quad (27)$$

With no compensation of the emission by the absorption, the electron should decay in a time

$$t = \frac{m_e c^2}{P} = 2.961 \times 10^{-20} \text{ [s]}. \qquad (28)$$

We have shown, that the power simultaneously absorbed and emitted in the dynamic equilibrium with the quantum vacuum is extremely high. Note that in the presence of an electrostatic field, the external energy density is different at the two sides of the electron . The electron radiates more energy towards the lower energy density region w.r.t. the other region with higher energy density. This explains the nature of the recoil force.

5. The hypothesis about the recoil nature of gravitational forces

Let us investigate, whether the above described nature of the electrostatic force as a recoil force applies also for gravitational forces. The attraction of two point masses corresponds the repel of two point charges of the same sign. This difference is caused by the fact, that the energy level of the vacuum in presence of the electrostatic field is higher than the energy level of the zero-field vacuum, while the gravitational field lowers the energy level of the vacuum. (see Appendix 3). However, the application of the ellipsoid model for the directivity pattern of the emission in the gravitational case requires the knowledge of the eccentricity of the ellipsoid. In the electrostatic case it was assumed that the eccentricity is $\varepsilon \approx |\vec{E}|/|\vec{E}_{\max}|$, where $E_{\max}$ is the maximum possible value of the electrostatic field strength postulated in [2]. The external field is always lower than $E_{\max}$, and usually $\varepsilon \ll 1$. Differently, the magnitude of eigen gravitational field at the border of the electron or any other elementary particle is very small and may be negligible in comparison to the magnitude of the external gravitational field. The maximum value of the electrostatic field is defined microscopicaaly as a field at the border of a charged particle. Differently, the eventual maximum value of the gravitational field is defined macroscopically, for example at the border of a neutron star or inside a black hole. Let us have an example with the neutron star PSR B1913 +16 (a member of a twin star). It has the mass $m_{PSR} = 1.441\times$ the mass of the sun $\approx 1.97 \times 10^{30}$ [kg] and a radius 20000 [m]. This yields the immense mass density $g = 8.469 \times 10^{16}$ [kg/m^3]. The corresponding magnitude of the gravitational field at the surface of the star is $\left|\vec{G}_{PSR}\right| = \frac{2gR_{PSR}}{3\gamma} = 9.49 \times 10^{11}$ [m/s^2]. A black hole having the mass equal to the earth-mass, should have a mass density 2×10^{30} [kg/m^3] and the Schwartzschild radius $r_S \approx 9 \times 10^{-3}$ [m]. This yields the gravitational field on the surface $\left|\vec{G}\right| = \frac{2gr_S}{3\gamma} = 1.08 \times 10^{19}$ [m/s^2]. We may assume that the eventual maximum possible value $\left|\vec{G}_{\max}\right|$ has the same order. The gravitational form of the Eq.(26) takes the form

$$m\left|\vec{G}\right| = \frac{\left|\vec{v}\right| P\varepsilon}{3c^2} \;\rightarrow\; P\varepsilon = \frac{3m\left|\vec{G}\right| c^2}{\left|\vec{v}\right|}. \qquad (29)$$

The calculation of the value of the power P would be possible if the eccentricity ε could be calculated. Actually we have not found a method enabling the determination of the value of ε. For a mass $m = 1$ [kG] located at the earth surface we get, using $\left|\vec{G}\right| = 9.81$ [m/s^2] and $\left|\vec{v}\right| = c$, $P\varepsilon \approx 8.83\times 10^9$ [W]. Of course, the power P is several order of larger, since certainly $\varepsilon \ll 1$. Note, that the power per a single particle is 6.025×10^{23} lower.

6. The radiation of the electron on the Bohr orbit

Last time the author found in reference [3–5] a short information, that the electron in the Bohr model of the hydrogen atom radiates electromagnetic energy and that this energy is simultaneously compensated by the vacuum. However, the author of [3–5] has not presented any evidence or calculations. As well he gave not a reference to the paper [1]. Let us quote a statement from the internet part of reference [3–5].

"There it is shown that the electron can be seen as continually radiating away its energy as predicted by classical theory, but simultaneously absorbing a compensating amount of energy from the ever-present sea of zero-point energy in which the atom is immersed, and an assumed equilibrium between these two processes leads to the correct values for the parameters known to define the ground-state orbit". End of citation.

In fact, the reading of the reference [3–5] supported our decision to write this paper. For completeness let us calculate the power of the electromagnetic radiation of the electron moving with a constant velocity ac (α – fine structure constant) on the circular orbit of the Bohr model of the hydrogen atom. In this example, there is no need to introduce corrections due to the finite mass of the proton and relativistic mass of the electron. With these assumptions the Bohr radius is given by the formula

$$r_B = \frac{\varepsilon_0}{\pi m_e}\left(\frac{h}{e}\right)^2. \tag{30}$$

The angular velocity of the electron is

$$\omega = \frac{\pi m_e e^4}{2\varepsilon_0^2 h^3}. \tag{31}$$

This yields the period of a single revolution

$$T = \frac{4\varepsilon_0^2 h^3}{m_e e^4}. \tag{32}$$

The instantaneous power radiated by a charge e moving along a path $s(t)$ with the acceleration $\ddot{s}(t)$ is derived in [6]

$$P(t) = \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\ddot{s}(t)\right)^2. \tag{33}$$

On the circular Bohr orbit we have a centrifugal time independent acceleration

$$\ddot{s} = r_B\omega^2 = \frac{\pi m_e e^6}{4\varepsilon_0^3 h^4}. \tag{34}$$

The insertion of (33) in (32) yields the following power radiated by the electron circulating on the Bohr orbit

$$P = \frac{\pi m_e^2 e^{14}}{96\varepsilon_0^7 c^3 h^8} = \frac{4\pi}{3}\frac{m_e^2 c^4}{h}\alpha^7 = 4.66936\times 10^{-8}\ [\mathrm{W}], \tag{35}$$

and the energy radiated during a single revolution is

$$E_T = PT = \frac{4\pi}{3} m_e c^2 \alpha^5. \tag{36}$$

We observe, that the power of the electromagnetic radiation of the electron in the Bohr model of the hydrogen atom is very small but finite. It is negligible w.r.t. the power due to the dynamic equilibrium of the electron with the quantum vacuum. Let us say, that we are aware that the Bohr model may be far from physical reality. For example, the author of [7] presented another model believing it is closer to the physical reality.

Appendix 1.

Derivation of the recoil force for an ellipsoidal power radiation pattern

We assume, that the angular power radiation pattern (power density per unit solid angle) is given by the rotation around the major axis of the ellipse

$$\sigma_\Omega = \sigma_{\max}\frac{1-\varepsilon^2}{1+\varepsilon\cos(\varphi)}\ [\mathrm{W/Ster}] \tag{A1}$$

where ε is the eccentricity of the ellipse. This formula uses the polar coordinates centred in the focus of the ellipsoid. The recoil force is given by the integral

$$\left|\vec{F}\right| = \frac{\left|\vec{v}\right|}{c}\int\limits_{4\pi}\sigma_\Omega\cdot\vec{n}_0 d\Omega \tag{A2}$$

where $\vec{v}$ is the velocity of radiation and $\vec{n}_0$ a unit vector directed along the major axis of the ellipse. The insertion of (A1) and using the projection of the radius centred in the focus on the major axis ($\cos(\varphi)$) yields

$$\left|\vec{F}\right| = \frac{\left|\vec{v}\right|}{c}\int\limits_{4\pi}\frac{\left(1-\varepsilon^2\right)\cos(\varphi)}{1+\varepsilon\cos(\varphi)}d\Omega. \tag{A3}$$

We get

$$\left|\vec{F}\right| = \frac{\sigma_{\max}}{c}\int\limits_0^{2\pi}\int\limits_0^{\pi}\frac{\left(1-\varepsilon^2\right)\cos(\varphi)\sin(\varphi)}{1+\varepsilon\cos(\varphi)}d\varphi d\psi. \tag{A4}$$

The evaluation of the integral yields

$$\left|\vec{F}\right| = \frac{\sigma_{\max}}{c}f_1(\varepsilon), \tag{A5}$$

where

$$f_1(\varepsilon) = \left[\frac{1-\varepsilon^2}{\varepsilon}\log\left(1-\varepsilon^2\right) + \left(1-\varepsilon^2\right)\sum_{n=1}^{\infty}\frac{\varepsilon^{2n-1}}{n(2n-1)}\right]2\pi. \tag{A6}$$

However, $\sigma_{\max}$ should be normalized to keep the total power P independent on ε. The power gain of the ellipsoid is given by the formula

$$G = \frac{4\pi}{B} \tag{A7}$$

where B is the equivalent solid angle

$$B = \int\limits_0^{2\pi}\int\limits_0^{\pi}\frac{1-\varepsilon^2}{1+\varepsilon\cos(\varphi)}\sin(\varphi)d\varphi d\psi = f_2(\varepsilon) \tag{A8}$$

with

$$f_2(\varepsilon) = \left[\left(1-\varepsilon^2\right)\log\left(\frac{1+\varepsilon}{1-\varepsilon}\right)\right]2\pi. \tag{A9}$$

Since $\sigma_{\max} = PG$ we get

$$\sigma_{\max} = \frac{P}{f_2(\varepsilon)}. \tag{A10}$$

The insertion of (A10) in (A5) yields

$$\left|\vec{F}\right| = \frac{P}{c}\frac{f_1(\varepsilon)}{f_2(\varepsilon)}. \tag{A11}$$

If $\varepsilon << 1$, the ratio $f_1(\varepsilon)/f_2(\varepsilon) \approx \varepsilon/3$. The versor $\vec{n}_0$ is in the direction of the x – axis. The angular directional pattern is defined w.r.t. the right focus. The recoil force has the direction opposite to the $x\ axis$.

Appendix 2.

The negative sign of the energy density of the gravitational field

In the frame of the analogies between electromagnetic and gravitation which apply for linearized Einstein's equations, the energy density of the gravitational field is given by the equation $E_G = -0.5\gamma\left|\vec{G}\right|^2$ [J/m^3], where $\vec{G}$ [m/s^2] is the gravitational field and $\gamma = 1.1927 \times 10^9$ [kg s^2/m^3] is the gravitational permittivity of free space [8]. Let us derive why the gravitational field lowers the energy density of the vacuum.

Consider two parallel infinite planes each covered by a mass density ρ_m [kg/m^2] (Fig. A1). The gravitational field inside the planes equals zero and its magnitude outside the plates is $\left|\vec{G}\right| = \rho_m/\gamma$. The energy density outside the planes is $E_G = -0.5\gamma\left|\vec{G}\right|^2 = -\rho_m^2/2\gamma$ [kg/ms^2]. Imagine that the distance between the planes is enlarged by Δz. The gravitational field in the volume defined by Δz is cancelled. Note that energy density and pressure have the same dimensions. Since the planes attract, the enlargement is a shift against the pressure and corresponds to the input of a positive energy. Therefore, the cancellation of the gravitational field requires a positive energy. In consequence, the energy of the gravitational field is negative. If we accept the hypothesis that energy is a positive defined quantity, the above negative energy corresponds to a lowering of the positive energy of the vacuum.

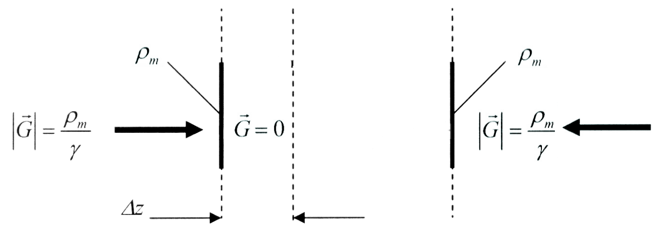

Fig. A1. Two parallel planes covered by a mass density ρ_m

REFERENCES

[1] S. Hahn, "Die Deutung der Kraft auf ein Elektron im Elektrostatischen Feld als Rűckstasskraft", *Kleinheubacher Berichte* 20, 145–149 (1976).

[2] R.L. Vallée, *L'Énergie Électromagnetique Matérielle et Gravitationelle*, Masson Cie, Paris, 1973.

[3] H.E. Puthoff, "Ground state of the hydrogen atom as a zero-point fluctuation state", *Phys.Rev.* 35 (10), 3266–3269 (1987).

[4] H.E. Puthoff, "Why atoms don't collapse", *New Scientist* 10, 26 (1987).

[5] H.E. Puthoff, "Quantum vacuum fluctuations: a new Rosetta stone of physics", http://www.Ido;phin.org/zpe.html (2007).

[6] K. Simonyi, *Theoretische Elektrotechnik*, VEB Verlag der Wissenschaften, Berlin, 1971.

[7] M. Gryziński, "The question of the atom", Homo Sapiens, Warszawa, 2001, (in Polish).

[8] R.L. Forward, "General relativity for the experimentalist", *Proc. IRE*, 892–904 (1961).