

# Finding of optimal excitation signal for testing of analog electronic circuits

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**Abstract.** This article presents combined approach to analog electronic circuits testing by means of evolutionary methods (genetic algorithms) and using some aspects of information theory utilisation and wavelet transformation. Purpose is to find optimal excitation signal, which maximises probability of fault detection and location. This paper focuses on most difficult case where very few (usually only input and output) nodes of integrated circuit under test are available.

**Key words:** fault detection, fault location, dictionary fault diagnosis, analog electronic circuits, wavelet transform, genetic algorithm.

## 1. Introduction

Fault diagnosis of analog electronic circuits is a difficult task, much more complex than testing of digital circuits. Deviation of parameter values (caused by design) is the main difficulty. Values of analog circuit components are not limited to nominal only, but lie within their tolerance ranges. This causes all measurable quantities also belong to some ranges for the same state of circuit (e.g. healthy or faulty). Fig.1 presents responses of a circuit (Fig.9) for unit step excitation [1]. The responses correspond to two different states of the circuit: healthy ( $F_0$ ) and faulty ( $F_7$ ; capacitor  $C_2$  100% above its tolerance range). Spread of the responses is caused by tolerance of components values. The responses overlap, which is the reason why state location is so difficult for analog circuits.

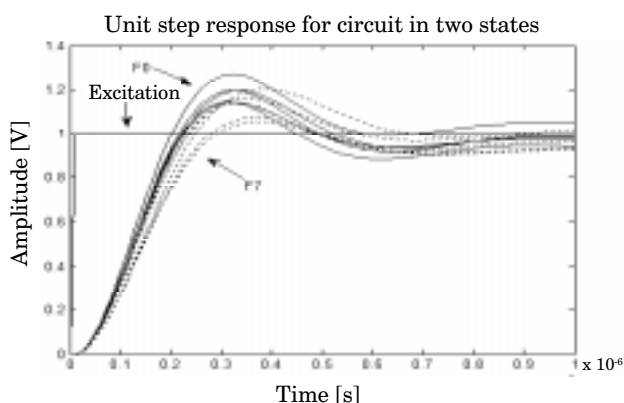


Fig. 1. Responses of circuit in two different states for unit step excitation

State of circuit can be defined in two manners. First one considers circuit functionality defined by design. Second one is based on implication that components within tolerance range ensure circuit to be healthy (circuit parameters

to be also within acceptable ranges). Result are two major classes of testing methods:

1) Specification Driven Testing (SDT) (or functional testing). Circuit under test (CUT) is validated only with respect to its functionality defined by design specifications [5].

There is set  $P$  of  $NP$  parameters  $p_i$  and each of them must lie within defined limits  $\langle p_i^{min}, p_i^{max} \rangle$ .

$$P = \{p_1, p_2, \dots, p_{N_p}\} \quad (1)$$

$$p_i \in \langle p_i^{min}, p_i^{max} \rangle, \quad i = 1, 2, \dots, N_p. \quad (2)$$

The parameters are defined by design (gain, bandwidth, power consumption etc.). Such approach treats circuit as single system, without necessity of knowledge about circuit internals ("black box"). However, this does not guarantee that internal components are within their tolerance ranges. The advantage of SDT is no need for CUT modelling. The disadvantages of SDT may be:

- complexity of measurements (large specification set, high cost, long measurement time, low accuracy)
- need for at-speed testing (sometimes technically challenging)
- in-system testing (not applicable when testing is damaging e.g. single-use systems)

2) Fault Driven Testing (FDT) is based on assumption that CUT satisfies design specifications, if values of its all  $N_C$  components lie within tolerance range. This, and inverse implication, may not be true in some cases. Value  $X_i$  of each component depends on its nominal value  $X_i^{nom}$  and absolute deviation  $\Delta X_i$  (or relative deviation – tolerance –  $tol_i$ ). Negative and positive deviations are assumed to be equal and monotonic. The deviations are result of manufacturing process.

$$X_i \in \langle X_i^{min}, X_i^{max} \rangle \quad (3)$$

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$$X_i^{\max} = X_i^{\text{nom}} + \Delta X_i = X_i^{\text{nom}} + X_i^{\text{nom}} \cdot \text{tol}_i = X_i^{\text{nom}} \cdot (1 + \text{tol}_i) \quad (4)$$

$$X_i^{\min} = X_i^{\text{nom}} - \Delta X_i = X_i^{\text{nom}} - X_i^{\text{nom}} \cdot \text{tol}_i = X_i^{\text{nom}} \cdot (1 - \text{tol}_i) \quad (5)$$

$$i = 1, 2, \dots, N_C$$

There are three goals of FDT:

- fault detection - recognition if CUT is "healthy" or faulty (test GO/NO GO).
- fault location - recognition which CUT component is faulty
- fault identification – recognition of faulty component and degree how much it is beyond its tolerance range [5]

In most cases fault location is more difficult than fault detection and simpler than fault identification. Symbolic presentation of successful faults detected, located and identified for some hypothetical diagnosis method is shown in the Fig.2.

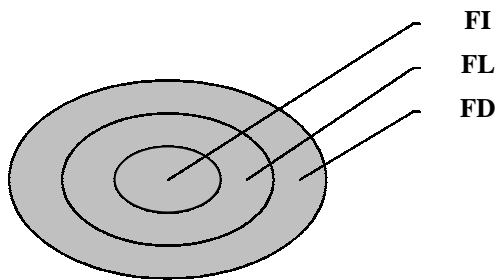


Fig. 2. Efficiency of fault diagnosis as result of chosen goal (FI – fault identification, FL – fault location, FD – fault detection)

FDT can be conducted in different conditions than normal CUT operation (e.g. filter tested by means of DC or aperiodic signal). This may be the advantage in case when in-system or at-speed measurements are expensive, difficult or impossible. The disadvantage is need of accurate CUT model.

Modelling of CUT is inseparable part of FDT. There can be distinguished two classes, depending when, in testing process, occurs mathematical analysis of CUT:

- 1) Simulation After Test (SAT). Mathematical analysis of CUT is performed after measurement. The goal is fault detection, location or identification based on measured response and CUT model. SAT requires large on-line ("during testing process") computational complexity, which is the greatest disadvantage. The advantage is no need for a priori definition of faults [5].
- 2) Simulation Before Test (SBT). CUT modelling is performed before measurement. CUT responses are compared with set of previously simulated responses. Faults are simulated before testing process (off-line) which is main. The disadvantage is a need of accurate circuit model and

set of a priori selected faults. Fig. 3 presents general division of testing methods [5].

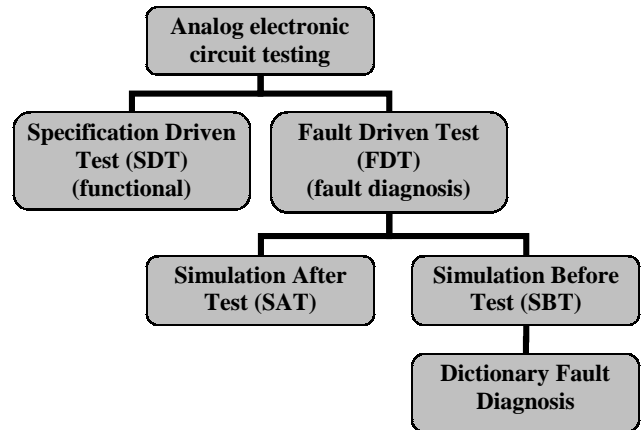


Fig. 3. Testing of analog electronic circuits

Dictionary fault diagnosis belongs to SBT method. It uses signatures of selected circuits as reference [5]. Signatures are usually responses of circuits with a priori selected faults. During testing process, response of CUT is compared with all signatures and then decision about state of the CUT is made. The simplest classification criterion is the most similarity between given signature and the CUT response.

Another division of fault diagnosis methods is based on type of signal (type of excitation) – Fig.4.

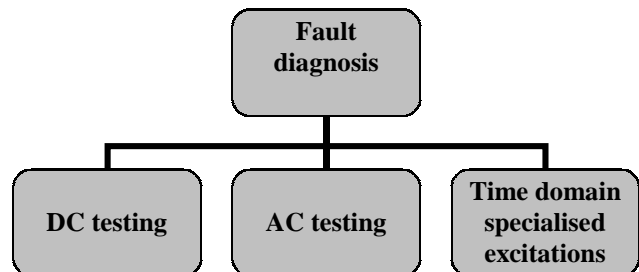


Fig. 4. Signals used in fault diagnosis of analog circuits

- 1) DC testing is the simplest method. Testing procedure is based on measurements of DC potentials at selected nodes or currents in selected branches. Simplicity of measurement instruments and short testing time are the advantages. The DC testing - except parametric and catastrophic faults of resistive elements - can only detect catastrophic faults of energy storage elements: short capacitors and open inductors.
- 2) AC testing uses single- or multi-frequency excitation. CUT response is analysed in time or frequency domain (magnitude and phase of appropriate frequency components). The method can be applied to circuits containing both resistive and energy storage components. The disadvantages are more complex measurement instruments and longer testing time as compared to DC testing.

3) Third type of testing uses specialised time domain aperiodic excitations. Fault analysis using such excitation is the most complex, but in many cases performs better than DC and AC testing. Response of circuit for aperiodic excitation may deliver more information about CUT, which is not always achievable with AC excitation, especially when only a few frequencies are used. Longer testing time and complexity of measurement instruments are main disadvantages.

A nowadays problem for electronic circuits testing is wide usage of integrated circuits that inhibit access to internal nodes. Generally, only input, output and supply current nodes (from power supply and to ground) are accessible. DC and AC testing may be helpless in such situation. Testability can be improved using complex aperiodic excitation applied to circuit input and analysis of response and supply current variations. It is also possible to have extra influence on CUT e.g. by modifying supply voltage. Testing in time domain by means of specialised signals is still the least popular comparing to DC and AC testing methods. It leads to following question: what should be a shape of the excitation signal to achieve the greatest possible fault detection and/or location and/or identification?

## 2. Description

This paper presents method of finding an optimal (or suboptimal) shape of time-domain excitation signal for purpose of a single fault detection and location. The presented work concerns only the case when the CUT works in linear range [1].

Excitation signal  $E$  is a vector of  $N_E$  discrete time samples  $e_i$ . The samples are spaced equally in time, with sampling period  $t_S$ :

$$E = \{e_1, e_2, \dots, e_{N_E}\} \quad (6)$$

$$t_{i+1}^e - t_i^e = t_S; \quad i = 1, 2, \dots, N_E - 1. \quad (7)$$

Such signal can be easily generated by means of digital-to-analog converter (DAC).

Selection of excitation sampling period  $t_S$  is very important. Too long time causes CUT to treat input signal as sequence of step functions – not as approximation of analog signal. Too short sampling time is also undesired. The excitation signal in its digitised form contains unnecessarily many samples. Chosen value is delay time of Dirac pulse response of examined  $2^{nd}$  order CUT ( $t_\delta^{nom}$ ). This seems to be sufficient compromise between amount of data to process and approximation quality of analog signal. Value of  $t_\delta^{nom}$  is easy to measure or simulate and has been taken from healthy circuit with all components having nominal value.

$$t_s = t_\delta^{nom}. \quad (8)$$

The testing procedure uses:

- genetic algorithm (GA) as a "search engine"
- information theory as descriptor of fault location and detection efficiency
- wavelet transform as a "feature extractor"

- fault dictionary used to "store" defined states (signatures) of the CUT.

GA is used as search engine in order to find values of excitation signal samples that maximises probability of fault detection and location. There is a population  $Pop$  containing  $N_{Ind}$  individuals  $Ind_i$ , each containing excitation signal encoded as vector  $E_i$  of digital samples  $e_j^i$ . Values of the samples are encoded as floating point numbers [4].

$$Pop = \{Ind_1, Ind_2, \dots, Ind_{N_{Ind}}\} \quad (9)$$

$$Ind_i = E_i = \{e_1^i, e_2^i, \dots, e_{N_E}^i\}; \quad i = 1, 2, \dots, N_{Ind}. \quad (10)$$

Main steps of the GA are presented in Fig. 5.

### 1) Initialisation

Initial shape of excitation held by the population (by all individuals) is the unit step.

$$\forall Ind_{i;i=1,2,\dots,N_{Ind}} = \begin{cases} e_1 = 0 \\ e_j = 1 \quad j = 2, 3, \dots, N_E \end{cases}. \quad (11)$$

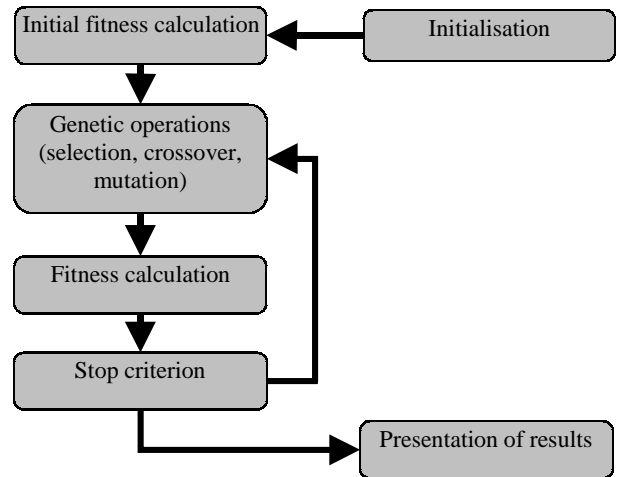


Fig. 5 Main steps of genetic algorithm

Initial fitness calculation computes fitness of every individual. This is required for genetic operations. Details of the fitness calculation are described in pt. 3.

### 2) Genetic operations.

The operation selection chooses individuals to become parents  $Pop^{par}$  for reproduction. Probability of selection depends on fitness value. The roulette method simulates a roulette wheel with the area proportional to fitness value of individual. Then a random number is used to select the individual with a probability equal to the area. Size (cardinality) of parent population  $Pop^{par}$  is equal to size of current population  $Pop$ .

$$Pop^{par} = select(Pop) \quad (12)$$

$$card(Pop^{par}) = card(Pop). \quad (13)$$

Process of reproduction creates offspring population  $Pop^{off}$  from the population of parents. Offspring contains three types of individuals created by means of following operations:

**Elite copying.** There is population  $Pop^{elite}$  of  $N_{Ind}^{elite}$  individuals that are directly copied from population of parents. The elite is small (10%) part of the offspring population.

They are chosen among  $N_{Ind}^{elite}$  parents having the highest value of fitness function.

$$N_{Ind}^{elite} = card(Pop^{elite}) = 0.1 \cdot card(Pop^{par}) \quad (14)$$

$$Ind_i^{elite} = \max(Pop); \quad i = 1, \dots, N_{Ind}^{elite} \quad (15)$$

**Crossover.** 80% of remaining part of offspring population is created by means of multi-point crossover. Samples of offspring excitation vector are chosen randomly from samples of corresponding two parents.

$$Pop^{cross} = cross(Pop^{par}). \quad (16)$$

**Mutation.** Remaining offspring is created from population of parents by means of mutation. The operation modifies single individual (excitation) by adding random Gaussian vector with 0 mean and standard deviation equal to 1 ( $G(0,1)$ ).

$$Pop^{mut} = mutate(Pop^{par}) \quad (17)$$

$$Ind_i^{mut} = Ind_i^{par} + G(0,1) \quad (18)$$

The offspring population is union of above subpopulations.

$$Pop^{off} = Pop^{elite} \cup Pop^{cross} \cup Pop^{mut} \quad (19)$$

Old population is replaced by the offspring population.

$$Pop = Pop^{off} \quad (20)$$

### 3) Fitness calculation

Fitness value of individual represents usefulness of corresponding excitation in process of fault detection or location. The process takes four steps presented in the Fig. 6.

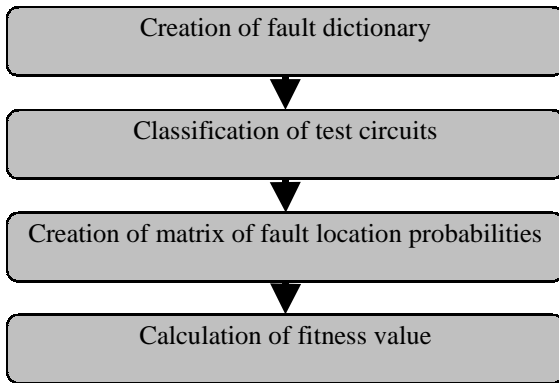


Fig. 6. Fitness calculation

**Creation of fault dictionary  $F_D$ .** There are defined  $N_F$  parametric faults  $F_i$  corresponding to chosen components. Non-faulty circuit is encoded as fault (state)  $F_0$ . Odd indexed faults encode values of components below

tolerance range. Evenly indexed faults encode values of components above tolerance range. Values of non-faulty elements are chosen randomly (uniformly) within their tolerance range. Value of relative tolerance  $tol_X$  is 5% for resistors and 10% for capacitors.

$$F_D = \{F_0, F_1, F_2, \dots, F_{N_F}\} \quad (21)$$

$$F_0 \rightarrow X_j \in < X_j^{\min}, X_j^{\max} >; \quad j = 1, \dots, N_C$$

$$F_i \rightarrow X_i = X_{nom} \cdot (1 + (-1)^i \cdot 2 \cdot tol_X) \quad (22)$$

$$i = 1, 2, 3, \dots, N_{F.}$$

The fault dictionary contains signatures  $S^i$  that are responses  $Y^i$  of reference CUT in the given state. Each signature (corresponding to the given fault) is averaged from 30 responses. The goal is to minimise spread of the responses caused by deviation of component values.

$$Y_j^i = \{y_{j,1}^i; y_{j,2}^i; \dots; y_{j,N_E}^i\} \quad (23)$$

where:

$i$  – fault number;  $i = 0, 1, \dots, N_F$

$j$  – average response number;  $j = 1, 2, \dots, 30$

$$S^i = \{s_1^i, s_2^i, \dots, s_{N_E}^i\} \quad (24)$$

$$S^i = average(Y_j^i); \quad j = 1, 2, \dots, 30$$

$$s_j^i = \frac{1}{30} \cdot \sum_{k=1}^{30} y_{k,j}^i \quad (25)$$

$$i = 0, 1, \dots, N_F; \quad j = 1, 2, \dots, N_E.$$

**Classification of test circuits.** There are created 100 test circuits for each fault. CUT in unknown state is classified to any defined state (fault) by comparing distance between measured CUT response and signatures stored in fault dictionary. The minimal distance criterion is used for classification. The distance between CUT response  $Y^{test}$  and each signature  $S^i$  is computed in two manners:

**Euclidean distance  $d_i^E$**

$$d_i^E = \sqrt{\sum_{k=1}^{N_E} (y_k^{test} - s_k^i)^2}; \quad i = 0, 1, \dots, N_F. \quad (26)$$

**Euclidean distance between wavelet coefficients  $G$ .** The continuous wavelet transform (CWT) is applied to measured CUT response  $Y^{test}$  and given signature  $S^i$ . Result is a finite set of coefficients  $G$ . The set is finite, because coefficients  $s$  (scale) and  $p$  (position) are chosen to have discrete values.

$$d_i^E = \sqrt{\sum_{k=1}^{N_E} (y_k^{test} - s_k^i)^2}; \quad i = 0, 1, \dots, N_F$$

$$G^{test}(s, p) = \int_{-\infty}^{\infty} Y^{test}(t) \cdot \psi_{s,p}(t) dt$$

$$G^{S^i}(s, p) = \int_{-\infty}^{\infty} S^i(t) \cdot \psi_{s,p}(t) dt \quad (27)$$

$$\psi_{s,p}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-p}{s}\right)$$

where:

$i$  – signature (fault) index;  $i=0,1,\dots,N_F$

$s$  – scale coefficient

$p$  – position coefficient

$\psi$  – wavelet function ("mother" wavelet)

The mother wavelet function is Daubechies 3<sup>rd</sup> order (db3) [2]. The Euclidean distance  $d^W$  is computed between wave-let coefficients  $G^{test}$  and  $G^S$ .

$$G^{test} = \begin{bmatrix} g_{11} & g_{12} & \dots \\ g_{21} & g_{22} & \dots \\ \dots & \dots & \dots \end{bmatrix}_{m \times n} \quad G^S = \begin{bmatrix} g_{11} & g_{12} & \dots \\ g_{21} & g_{22} & \dots \\ \dots & \dots & \dots \end{bmatrix}_{m \times n} \quad (28)$$

$$d_i^W = \left| \sum_{c=1}^m \sum_{r=1}^n m_{rc} \right| \quad i = 1, 2, \dots, N_F$$

The wavelet transform is used to enhance differences between responses of healthy and faulty CUT. Utilisation of spectral as well as time domain information (impossible for Fourier transform) is great advantage of wavelet transform. The transform is able to bring out features of analysed signal that can be difficult to detect by means of separate time or frequency analysis.

**Creation of matrix of fault location probabilities.** Information theory is utilised in order to evaluate efficiency of circuit states separation. Model of testing process is based on two-symbol information source and lossy information channel (Fig.7), where  $p_{ij}$  is probability of symbol  $j$  reception, if symbol  $i$  has been transmitted [3]. The probabilities  $p_{ij}$  are calculated as ratio of number of circuits classified to state  $D_j$  to number of circuits in state  $F_i$ :

$$p_{ij} = \frac{D_j}{F_i} \quad (29)$$

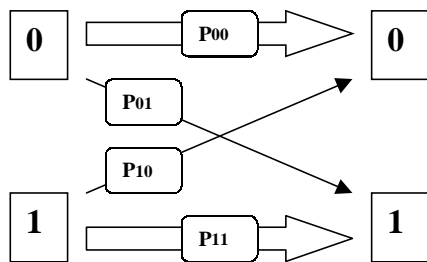


Fig. 7. Binary lossy information channel

The probabilities  $p_{ij}$  can be written in matrix of fault location probabilities ( $M^L$ ):

	$D_0$	$D_1$	$D_2$	...	$D_N$
$F_0$	$P_{00}$	$P_{01}$	$P_{02}$	...	$P_{0N}$
$F_1$	$P_{10}$	$P_{11}$	$P_{12}$	...	$P_{1N}$
$F_2$	$P_{20}$	$P_{21}$	$P_{22}$	...	$P_{2N}$
...	...	...	...	...	...
$F_N$	$P_{N0}$	$P_{N1}$	$P_{N2}$	...	$P_{NN}$

where:

$F_0$  – healthy circuit

$F_i$  – faulty circuit in  $i$ -th state

$D_i$  – decision: circuit in  $i$ -th state

$P_{ij}$  – probability that circuit in  $i$ -th state is classified to  $j$ -th state

$$M^L = \begin{bmatrix} m_{11} & m_{12} & \dots \\ m_{21} & m_{22} & \dots \\ \dots & \dots & \dots \end{bmatrix}_{N_F+1 \times N_F+1} \quad (30)$$

Main diagonal of the matrix contains probabilities of correct classifications. Used optimisation criterion is to obtain values at main diagonal as close 1 as possible. This implies values outside main diagonal as close 0 as possible.

In case of fault detection (test GO/NO GO), probabilities of fault detection can be written as follows:

	$D_h$	$D_f$
healthy	$P_{hh}$	$P_{hf}$
faulty	$P_{fh}$	$P_{ff}$

where:

$D_h$  – decision: circuit healthy

$D_f$  – decision: circuit faulty

$P_{hh}$  – probability of correct classification of healthy circuit

$P_{ff}$  – probability of correct classification of faulty circuit

$P_{fh}$  – probability of classification of faulty circuit as healthy one

$P_{hf}$  – probability of classification of healthy circuit as faulty one

The probabilities of fault detection can be found empirically, but in order to avoid time consuming simulations of CUTs, above values can be computed by means of the following formulas:

$$p_{pp} = p_{00} \quad (31)$$

$$p_{fp} = \frac{1}{N} \sum_{i=1}^N p_{i0} \quad (32)$$

$$p_{pf} = \sum_{i=1}^N p_{0i} = 1 - p_{00} = 1 - p_{pp} \quad (33)$$

$$p_{ff} = \frac{1}{N} \sum_{j=1}^N \sum_{i=1}^N p_{ij} \quad (34)$$

**Calculation of fitness value.** The fitness value  $fit$  of found individual (excitation) is sum of probabilities of correct fault location (sum of entries on main diagonal).

$$fit = \sum_{i=1}^{N_F} m_{ii}^L \quad (35)$$

The GA may also take into consideration frequency spectrum of found excitation. This promotes solutions with narrower bandwidth (less high frequency components). The spectrum is divided into two equal bands: low and high. Afterwards, fitness value  $fit$  of the given solution is modified, according to power spectral density ( $P_{low}$  and  $P_{high}$ ) in appropriate band.

$$low = \langle 0; f_{mid} \rangle \quad high = \langle f_{mid}; f_{max} \rangle$$

$$f_{max} = \frac{1}{2} \cdot \frac{1}{t_s} \quad f_{mid} = \frac{1}{2} \cdot f_{max} \quad (36)$$

$$fit = \begin{cases} fit \cdot 2 \leftarrow P_{low} > P_{high} \\ fit \cdot \frac{1}{2} \leftarrow P_{low} \leq P_{high} \end{cases} \quad (37)$$

4) Stop criterion

There is local best individual found after each iteration of the GA (Fig. 8). Chosen stop criterion of the GA is lack of improvement of found best solution. The GA is stopped if fitness  $fit_i^{best}$  of current best solution is not improved for more than 10 iterations i.

$$stop \text{ if } fit_i^{best} \leq fit_{i-10}^{best} \quad (38)$$

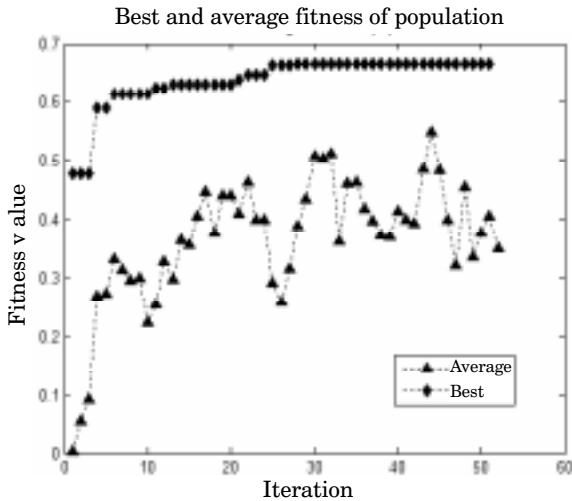


Fig. 8. Example of best and average values of fitness

3. Example and results

The fault diagnosis methods have been verified using a low pass filter [1] (Fig. 9). Fault dictionary contains signatures of selected eight parametric faults corresponding to four chosen components ( $C_1, C_2, R_2, R_4$ ) of the circuit [1]. Non – faulty circuit is represented by state  $F_0$ , other faults are defined in Table1.

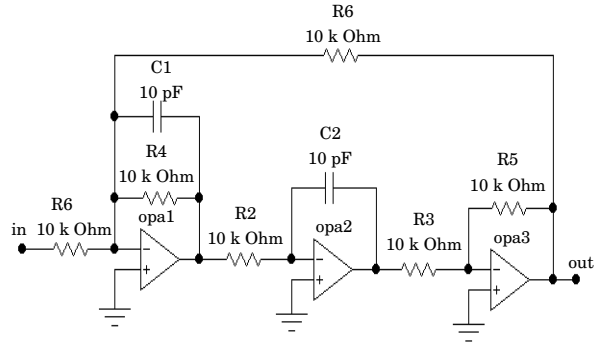


Fig. 9. Schematic of the low pass filter after Ref. 1

Table 1  
Faults and corresponding components

F	
$F_0$	-
$F_1$	$R_2$ "small"
$F_2$	$R_2$ "large"
$F_3$	$R_4$ "small"
$F_4$	$R_4$ "large"
$F_5$	$C_1$ "small"
$F_6$	$C_1$ "large"
$F_7$	$C_2$ "small"
$F_8$	$C_2$ "large"

Population for GA had 30 individuals. The excitation vector  $E$  has been chosen to be 100 and 200 samples long. Required resolution of spectral analysis was the objective. The method has been verified on three cases, which differed by feature extraction of CUT response and spectrum optimisation of found excitation:

1) Euclidean distance classifier. Number of GA iterations: 113. Best solution is presented in the Fig. 10 and its spectrum in Fig. 11. The spectrum has been computed over whole signal length. Probabilities of fault location and detection for both unit step and obtained excitation are presented in Tables 2-5. Average computation time took 19 hours.

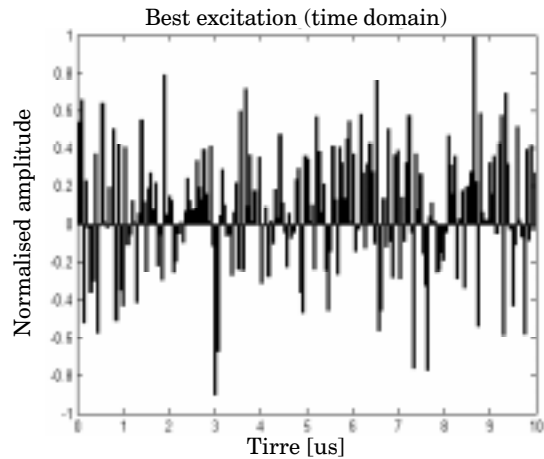


Fig. 10. Optimal excitation (time domain)

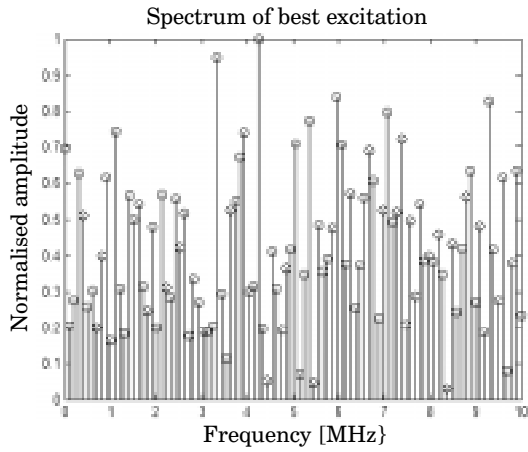


Fig. 11. Spectrum of optimal excitation

Table 2  
Probability of fault classification for optimal excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>25</b>	22	19	7	13	<u>2</u>	8	1	<u>3</u>
F1	4	<b>32</b>	2	3	29	<u>3</u>	0	27	<u>0</u>
F2	27	4	<b>36</b>	10	1	<u>0</u>	5	0	17
F3	7	5	23	<b>47</b>	0	<u>4</u>	0	0	14
F4	13	14	4	0	<b>43</b>	<u>0</u>	18	8	<u>0</u>
F5	0	1	0	14	<u>0</u>	<b>73</b>	0	12	<u>0</u>
F6	<u>2</u>	0	1	0	8	<u>0</u>	<b>71</b>	0	<u>18</u>
F7	0	17	0	0	<u>3</u>	<u>0</u>	0	<b>80</b>	<u>0</u>
F8	2	0	19	2	0	<u>0</u>	5	0	<b>72</b>

Table 3  
Probability of fault classification for unit step excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>3</b>	8	3	3	18	35	0	2	28
F1	2	<b>14</b>	1	0	19	29	1	12	22
F2	8	0	<b>13</b>	3	3	37	2	0	34
F3	4	0	11	<b>12</b>	4	35	0	0	34
F4	8	15	2	0	<b>28</b>	20	3	3	21
F5	0	0	0	6	14	<b>41</b>	0	2	37
F6	12	4	6	0	5	21	<b>15</b>	0	37
F7	0	10	0	0	20	27	0	<b>27</b>	16
F8	2	0	21	4	0	23	1	0	<b>49</b>

Table 4  
Probability of fault detection for optimal excitation

	D <sub>H</sub>	D <sub>F</sub>
H	<b>25%</b>	<b>75%</b>
F	7%	<b>93%</b>

Table 5  
Probability of fault detection for unit step excitation

	D <sub>H</sub>	D <sub>F</sub>
H	<b>3%</b>	97%
F	4%	<b>96%</b>

2) Euclidean-wavelet distance classifier. Number of GA iterations: 40. Base wavelet: Daubechies, 3<sup>rd</sup> order (db3). Average computation time: 13 hours.

Table 6  
Probability of fault classification for optimal excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>34</b>	<u>16</u>	14	9	<u>12</u>	3	7	2	3
F1	19	<b>29</b>	1	0	25	3	2	21	0
F2	16	<u>1</u>	<b>26</b>	12	<u>2</u>	1	13	0	29
F3	15	<u>3</u>	16	<b>36</b>	<u>2</u>	14	0	0	14
F4	16	<u>21</u>	2	0	<b>42</b>	0	14	5	0
F5	1	<u>0</u>	0	13	<u>0</u>	<b>82</b>	0	4	0
F6	3	<u>0</u>	0	0	<u>12</u>	0	<b>74</b>	0	11
F7	0	<u>22</u>	0	0	1	1	0	<b>76</b>	0
F8	1	<u>0</u>	23	<u>1</u>	<u>0</u>	0	3	0	<b>72</b>

Table 7  
Probability of fault classification for unit step excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>1</b>	44	3	2	44	1	0	3	2
F1	6	<b>48</b>	0	2	24	2	0	18	0
F2	4	24	<b>6</b>	13	31	0	1	0	21
F3	2	31	6	<b>16</b>	31	2	0	0	12
F4	3	55	0	1	<b>29</b>	0	4	6	2
F5	0	41	0	8	28	<b>13</b>	0	10	0
F6	4	24	0	4	32	0	<b>23</b>	0	13
F7	2	47	0	0	2	2	0	<b>47</b>	0
F8	0	14	5	23	25	0	0	0	<b>33</b>

Table 8  
Probability of fault detection for optimal excitation

	D <sub>H</sub>	D <sub>F</sub>
H	<b>34%</b>	<b>66%</b>
F	9%	<b>91%</b>

Table 9  
Probability of fault detection for unit step excitation

	D <sub>H</sub>	D <sub>F</sub>
H	<b>1%</b>	99%
F	3%	<b>97%</b>

3) Euclidean distance classifier with spectrum optimisation of found excitation. Number of GA iterations: 61. Average computation time: 12 hours.

Table 10  
Probability of fault classification for optimal excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>17</b>	12	12	21	23	6	4	3	<u>2</u>
F1	4	<b>35</b>	0	10	<u>14</u>	7	0	30	<u>0</u>
F2	16	1	<b>24</b>	16	7	1	7	0	<b>28</b>
F3	28	4	19	<b>20</b>	14	2	6	0	<u>7</u>
F4	9	33	2	17	<b>15</b>	5	1	18	<u>0</u>
F5	4	32	0	18	23	<b>11</b>	0	12	<u>0</u>
F6	15	1	15	16	13	3	<b>6</b>	0	31
F7	0	8	0	0	<u>0</u>	0	0	<b>92</b>	<u>0</u>
F8	3	0	17	<u>0</u>	0	0	13	0	<b>67</b>

Table 11  
Probability of fault classification for unit step excitation [%]

	D0	D1	D2	D3	D4	D5	D6	D7	D8
F0	<b>16</b>	9	0	19	18	1	6	1	30
F1	11	<b>16</b>	2	10	36	4	7	4	10
F2	9	10	<b>0</b>	15	9	2	5	5	45
F3	10	7	2	<b>28</b>	8	0	4	1	40
F4	12	12	1	11	<b>37</b>	4	2	5	16
F5	12	12	6	19	14	<b>0</b>	6	5	26
F6	10	11	2	25	8	6	<b>6</b>	2	30
F7	5	24	3	4	52	1	1	<b>9</b>	1
F8	8	5	3	23	0	1	3	0	<b>57</b>

Table 12  
Probability of fault detection for optimal excitation

	Dp	Df
P	<b>17.0%</b>	<b>83.0%</b>
F	<b>9.9%</b>	<b>90.1%</b>

Table 13  
Probability of fault detection for unit step excitation

	Dp	Df
P	<b>16.0%</b>	84.0%
F	<b>9.6%</b>	<b>90.4%</b>

By comparing results contained in tables, the following conclusions can be drawn:

a) case 1 significantly increased probability of correct fault location (Tables 2 and 3; main diagonals) and decreased risk of the incorrect fault location (Tab. 2; underscored entries outside main diagonal)

b) optimal excitation found in case 2 is very similar to the one found in case 1 (both signal shape in time domain and spectrum). The elimination of high frequency components has not been achieved, which leads to conclusion that excitations with limited bandwidth have poor fault location and detection abilities. Such signals are missing in the final solution, because they have not "survived" the evolution process

c) utilisation of wavelet transform (case 3) returned excitation signal shape and bandwidth similar to signals found by previous cases. Additionally, wavelet analysis has interesting and desired feature to localise faults which are completely undetectable by classifiers based on Euclidean distance only (case 1 and 2) (Tables 10 and 11; *F2-D2 and F5-D5*)

Average computation time took 12 - 19 hours. Computational complexity mainly depends on calculation of the fitness function in the GA process. The most of computer resources have been taken by simulation of the analog electronic circuits (PSpice simulator). Total time of GA work is strongly related with number of GA iterations, which is theoretically unpredictable.

### Conclusions

The presented method has proven that fault detection and location can be significantly improved by means of testing using specialised aperiodic signals. The results have been compared with fault analysis using the simplest aperiodic signal: a unit step. It has been also observed that wavelet analysis makes possible to locate (and detect) single faults completely undetectable by means of testing using unit step excitation. Importance of high frequency spectrum components is another conclusion. Excitations with limited spectrum are missing in the final solution, which suggests they have not "survived" the evolution process. The reason may be poor fault location and detection abilities of such signals.

The presented approach has disadvantages too. Great computational complexity is the most important. However, the method belongs to SBT class of testing procedures, so computational time is not as crucial as in case of SAT procedure.

The method is in early development stage and requires further work. Main directions are reduction of computational effort and improvement of fault location and detection, including cases of multiple faults. Other types of circuits (also nonlinear) will be examined with the use of the method.

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