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Electrokinetics in random piezoelectric porous media

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Abstract. Macroscopic coefficients together with a Darcy law are obtained for porous piezoelectric medium with random, not necessarily ergodic, distribution of pores in which a two-ionic electrolyte flows. Peculiarities of stochastic porosity are indicated.

Key words: homogenisation, ergodicity flow.

1. Introduction

In an analysis of flows through porous media one deals both, with deterministic and stochastic media. Many porous media, both natural ones as well as man-made reveal random distribution of pores. The synthetic article [1] provides an account of effective models of flows through random rigid porous media (transport problem). Electrokinetical phenomena in such media were studied by Adler et al [2].

In this paper the problem of stationary flow of two-ionic species electrolyte through random piezoelectric porous media is studied, thus extending our earlier paper [3], where spatial periodicity of porous medium was assumed. To derive the macroscopic equations we use the method od stochastic two-scale convergence in the mean developed by Bourgeat et al. [4].

Solid phase was assumed to be piezoelectric since according to [5] wet bone reveals piezoelectric properties, cf. also [6]. We recall that a strong conviction prevails that for electric effects in bone only streaming potentials are responsible.

Macroscopic equations are given in Section 4 without the assumption of ergodicity. In Section 5 we provide comments on the case where ergodicity applies.

2. Description of random porous media and the method of stochastic two-scale convergence in the mean

Natural and man-made porous media usually possess formidably complex microstructure, often hierarchical. In this paper we shall not discuss hierarchical microstructures revealed, for instance by fractured porous media and biological tissues like bone and soft tissue. However, recently developed stochastic reiterated homogenisation enables one to determine macroscopic properties of random porous media with hierarchical architecture [7].

Let $(\Omega, \mathcal{F}, \mu)$ denote a probability space where $\mathcal{F}$ is a complete σ -algebra and μ is the probabilistic measure. Assume that Ω is acted on by an n-dimensional dynamical system $T(\mathbf{x}) : \Omega \to \Omega$, such that for each $\mathbf{x} \in I\!R^n$, both $T(\mathbf{x})$ and $T(\mathbf{x})^{-1}$ are measurable, and such that the following conditions are satisfied: (a) $T(\mathbf{0})$ is the identity map on Ω and for $\mathbf{x}_1, \mathbf{x}_2 \in I\!R^n$, $T(\mathbf{x}_1+\mathbf{x}_2) = T(\mathbf{x}_1)T(\mathbf{x}_2)$; (b) for each $\mathbf{x} \in I\!R^n$ and measurable set $F \in \mathcal{F}$, $\mu(T(\mathbf{x})^{-1}F) = \mu(F)$, i.e. μ is an invariant measure for T; (c) or each $F \in \mathcal{F}$, the set $\{(\mathbf{x}, \omega) \in I\!R^n \times \Omega | T(\mathbf{x})\omega \in F\}$ is a $d\mathbf{x} \times d\mu$ measurable subset of $I\!R^n \times \Omega$, where $d\mathbf{x}$ stands for the Lebesgue measure on $I\!R^n$, cf. [4].

We observe that $T(\mathbf{x})^{-1} = T(-\mathbf{x})$. The dynamical system satisfying (a)-(c) is also called a measure preserving flow. We introduce random homogeneous fields, starting from the random variable f:

$$f \in L^1(\Omega), \quad \tilde{f}(\mathbf{x}, \omega) \equiv f(T(\mathbf{x})\omega). \tag{1}$$

We observe that $\tilde{f}$ is also called the statistically homogeneous (i.e. stationary) random process. Statistical homogeneity means that two geometric points of the space are statistically undistinguishable, or the statistical properties of the medium are invariant under the action of translation. Then we have a group $\{U\mathbf{x} | \mathbf{x} \in I\!R^n\}$ of isometries on $L^2(\Omega) = L^2(\Omega, \mathcal{F}, \mu)$ defined by

$$\left(U(\mathbf{x})f\right)(\omega) = f\left(T(\mathbf{x})\omega\right), \quad \mathbf{x} \in I\!R^n, \omega \in \Omega, f \in L^2(\Omega).$$

A dynamical system is said to be ergodic, if every invariant function, i.e. satisfying $f(T(\mathbf{x})\omega) = f(\omega)$ is constant almost everywhere in Ω.

Examples of statistically homogeneous media are provided in [1, 7, 8].

Let $g \in L^1_{\text{loc}}(I\!R^n)$, i.e. g is integrable on every measurable bounded set $K \subset I\!R^n$. A number $M\{g\}$ is called the mean value of g if

$$\lim_{\varepsilon \to 0} \int_K g(\varepsilon^{-1}\mathbf{x}) d\mathbf{x} = |K| \, M\{g\}. \tag{2}$$

Here $|K|$ denotes the Lebesgue measure of K. Of crucial importance is the Birkhoff ergodic theorem which states

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that for $f \in L^\alpha(\Omega), \alpha \geq 1$,

$$f(T(\frac{\mathbf{x}}{\varepsilon})\omega) \rightharpoonup M\{f(T(\mathbf{x})\omega)\} \quad \text{weakly in } L^\alpha_{\text{loc}} \tag{3}$$

and $M\{f(T(\mathbf{x})\omega)\}$, considered as a function of $\omega \in \Omega$, is invariant. Moreover, we have

$$\langle f\rangle \overset{\text{df}}{=} \int_\Omega f(\omega)d\mu = \int_\Omega M\{f(T(\mathbf{x})\omega)\}d\mu. \tag{4}$$

In particular, if the system $T(\mathbf{x})$ is ergodic, then

$$M\{f(T(\mathbf{x})\omega)\} = \langle f\rangle \qquad \text{for almost all } \omega \in \Omega.$$

Let Q be a given, deterministic, bounded domain in $I\!R^n$ and let $G \in \mathcal{F}$. We set

$$G(\omega) = \{\mathbf{x} \in I\!R^n | T(\mathbf{x})\omega \in G\}, \tag{5}$$

$$\begin{gathered} Q_\varepsilon(\omega) = Q \setminus \mathcal{G}_\varepsilon(\omega), \quad \text{where} \\ \mathcal{G}_\varepsilon(\omega) = \{\mathbf{x} \in I\!R^n | \varepsilon^{-1}\mathbf{x} \in G(\omega)\}. \end{gathered} \tag{6}$$

Such a definition of random domain $Q_\varepsilon(\omega)$ is suitable for theoretical considerations. In practice, the random sets $G(\omega)$ or $G_\varepsilon(\omega)$ have to be described more precisely, cf. [1,7,9] and the references therein.

To carry out stochastic homogenisation, elements of local stochastic calculus are needed. For more details, the reader is referred to [1,4,7].

Anyway, one can define the stochastic gradient $\nabla_\omega f$, stochastic divergence $\text{div}_\omega \mathbf{v}$, etc.

In the periodic case ω is to be identified with local variable $\mathbf{y} \in Y$, where Y is the so-called basic cell.

The set of all functions $f \in L^2(\Omega)$ invariant for T (i.e. $f(T(\mathbf{x})) = f, \mu$ – a.e. on Ω, for all $\mathbf{x} \in I\!R^n$) is a closed subset of $L^2(\Omega)$ and denoted by $I^2(\Omega)$. We set $M^2(\Omega) = [I^2(\Omega)]^\perp$. We introduce a projection $E : L^2(\Omega) \to L^2(\Omega)$ determined by

$$\begin{gathered} (Ef)(\omega) = \lim_{\lambda\to\infty} \frac{1}{(2\lambda)^n} \int_{[-\lambda,\lambda]^n} f(T(\mathbf{x})\omega)d\mathbf{x}, \\ \mu \; - \; \text{a.e. } \omega \in \Omega. \end{gathered} \tag{7}$$

We have $M^2(\Omega) = \ker E$; moreover: (i) if $f \in L^2(\Omega)$ then $f \in I^2(\Omega)$ if and only if $\nabla_\omega f = 0$, (ii) for any multi-index $\alpha = (\alpha_1, \ldots, \alpha_n)$,

$$\partial^\alpha \varphi(T(\mathbf{x})\omega) = (D^\alpha\varphi)(T(\mathbf{x})\omega), \quad \varphi \in \mathcal{D}^\infty(\Omega)$$

where $\partial^\alpha = \partial^{|\alpha|}/\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}$, (iii) let $\mathbf{u} \in L^2(\Omega)^n$, $\mathbf{v} \in L^2(\Omega)^n$, $\text{curl}_\omega \mathbf{u} = \mathbf{0}$, $\text{div}_\omega \mathbf{v} = 0$, then

$$\int_\Omega \mathbf{u}\cdot\mathbf{v}\,d\mu = \int_\Omega E(\mathbf{u})\cdot E(\mathbf{v})\,d\mu. \tag{8}$$

Furthermore, if T is ergodic then (8) yields an extension of the Hill-type relation:

$$\int_\Omega \mathbf{u}\cdot\mathbf{v}\,d\mu = \int_\Omega \mathbf{u}\,d\mu \cdot \int_\Omega \mathbf{v}\,d\mu. \tag{9}$$

Now we are in a position to introduce the fundamental notion.

DEFINITION 1. A sequence $\{u^\varepsilon\}_{\varepsilon>0}$ in $L(Q\times\Omega)$ is said to stochastically two-scale converge in the mean to $u \in L^2(Q\times\Omega)$ if for all $\psi \in L(Q\times\Omega)$

$$\begin{aligned} &\lim_{\varepsilon\to 0} \int_{Q\times\Omega} u^\varepsilon(\mathbf{x},\omega)\psi(\mathbf{x}, T(\varepsilon^{-1}\mathbf{x})\omega)d\mathbf{x}d\mu \\ &= \int_{Q\times\Omega} u(\mathbf{x},\omega)\psi(\mathbf{x},\omega)d\mathbf{x}d\mu. \end{aligned} \tag{10}$$

The properties of stochastically two-scale convergent sequences like $\{u^\varepsilon\}_{\varepsilon>0}$ and $\{\varepsilon\nabla u^\varepsilon\}$ are studied in [4], cf. also [7]. These properties will be exploited in Section 4 of the present paper. We also need to extend the mapping E in order to cope with the so-called stochastic nonuniform homogenisation. To this end for each $\mathbf{y} \in I\!R^n$ we define the mapping $\tilde{T}(\mathbf{y}) : Q\times\Omega \to Q\times\Omega$ by $\tilde{T}(\mathbf{y})(\mathbf{x},\omega) = (\mathbf{x}, T(\mathbf{y})\omega)$. We observe that $\{\tilde{T}(\mathbf{y}) | \mathbf{y} \in I\!R^n\}$ is an n-dimensional dynamical system on $Q\times\Omega$. Replacing Ω, T by $(Q\times\Omega, \tilde{T})$ we extend (7) as follows

$$\tilde{T}g(\mathbf{x},\omega) = E[g(\mathbf{x},\cdot)](\omega)$$

or

$$\tilde{E}g(\mathbf{x},\omega) = \lim_{\lambda\to\infty} \frac{1}{(2\lambda)^n} \int_{[-\lambda,\lambda]^n} g(\mathbf{x}, T(\mathbf{y})\omega)d\mathbf{y} \tag{11}$$

$E\tilde{g}$ does not depend on $\omega \in \Omega$ (μ - a.e.) provided that μ is ergodic for T.

3. Equations of flow of electrolyte through piezoelectric random porous medium

Let $Q^s_\varepsilon(\omega) = Q \setminus \overline{Q}_\varepsilon(\omega)$ and $Q^\ell_\varepsilon(\omega) = Q \setminus \overline{Q}^s_\varepsilon(\omega)$, where $\varepsilon > 0$ is a small parameter characterizing microstructure. We assume that the sets $Q^\ell_\varepsilon(\omega)$ are connected. By $\mathbf{u}^\varepsilon(t,\mathbf{x},\omega)$ and $\mathbf{v}^\varepsilon(t,\mathbf{x},\omega)$ we denote fields of displacement in the piezoelectric phase $Q^s_\varepsilon(\omega)$ and velocity in the fluid-ionic phase $Q^\ell_\varepsilon(\omega)$, respectively. The pressure field, volume density of positive (negative) ions, and the corresponding current vectors are denoted by $p^\varepsilon(t,\mathbf{x},\omega)$, $q^{(+)^\varepsilon}(t,\mathbf{x},\omega)$, $q^{(-)^\varepsilon}(t,\mathbf{x},\omega)$, and $\mathbf{J}^{(\pm)\varepsilon}(t,\mathbf{x},\omega)$, respectively. Obviously, t stands for the time variable, $t \in [0,\tau]$. By $\Phi^\varepsilon(t,\mathbf{x},\omega)$ we denote the electric potential field.

The set of equations for the fields $\mathbf{u}^\varepsilon, \mathbf{v}^\varepsilon, p^\varepsilon, \Phi^\varepsilon, q^{(\pm)^\varepsilon}$, and $\mathbf{J}^{(\pm)\varepsilon}$ assume the following form:
– in the solid piezoelectric phase $Q^s_\varepsilon(\omega)$

$$\begin{aligned} &\rho^s\ddot{\mathbf{u}}^\varepsilon = \text{div}_\mathbf{x}[\mathbf{a}^\varepsilon\mathbf{e}(\mathbf{u}^\varepsilon) - \boldsymbol{\pi}^{\varepsilon(*)}\mathbf{E}(\Phi^\varepsilon)], \\ &\text{div}_\mathbf{x}[\boldsymbol{\pi}^\varepsilon\mathbf{e}(\mathbf{u}^\varepsilon) + [\boldsymbol{\epsilon}^{s\varepsilon}\mathbf{E}(\Phi^\varepsilon)] = 0 \end{aligned} \tag{12}$$

where

$$\mathbf{a}^\varepsilon(\mathbf{x},\omega) = \mathbf{a}(\mathbf{x}, T(\varepsilon^{-1}\mathbf{x})\omega),$$

$$\boldsymbol{\pi}^\varepsilon(\mathbf{x},\omega) = \boldsymbol{\pi}(\mathbf{x}, T(\varepsilon^{-1}\mathbf{x})\omega),$$

$$\boldsymbol{\epsilon}^{s\varepsilon}(\mathbf{x},\omega) = \boldsymbol{\epsilon}(\mathbf{x}, T(\varepsilon^{-1}\mathbf{x})\omega),$$

$$\mathbf{E}_\mathbf{x}(\Phi^\varepsilon) = -\nabla_\mathbf{x}\Phi^\varepsilon,$$

$$\Big(\boldsymbol{\pi}^{\varepsilon(*)}\mathbf{E}_\mathbf{x}(\Phi^\varepsilon)\Big)_{ij} = -\,\pi^\varepsilon_{kij}\frac{\partial\Phi^\varepsilon}{\partial x_k}.$$

Here $\left(\epsilon_{ij}^{s\varepsilon}\right)=\left(\epsilon_{ij}^{s\varepsilon}(\mathbf{x},\omega)\right)$ is the matrix of dielectric moduli in the solid phase, $\mathbf{e}(\mathbf{u})$ stands for the small strain tensor, and

$$\dot{\mathbf{u}}=\frac{\partial \mathbf{u}}{\partial t};$$

– in the fluid-ionic phase $Q_\varepsilon^\ell(\omega)$

$$\begin{aligned}
&\rho^\ell \dot{\mathbf{v}}^\varepsilon=\varepsilon^2\eta\Delta_{\mathbf{x}}(\mathbf{v}^\varepsilon)-\nabla_{\mathbf{x}}p^\varepsilon+\mathbf{f}^g+q^\varepsilon\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon)-\kappa\nabla_{\mathbf{x}}q^\varepsilon,\\
&\mathrm{div}_{\mathbf{x}}\mathbf{v}^\varepsilon=0,\\
&\mathrm{div}_{\mathbf{x}}\left(\boldsymbol{\epsilon}^{\ell\varepsilon}\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon)\right)=q^\varepsilon,\\
&\frac{\partial q^\pm}{\partial t}+\mathrm{div}_{\mathbf{x}}\mathbf{J}^{(\pm)\varepsilon}=0,\qquad q^\varepsilon=q^{(+)\varepsilon}+q^{(-)\varepsilon}.
\end{aligned}\tag{13}$$

More precisely, (12) holds in $(0,\tau)\times Q_\varepsilon^s(\omega)$ while (13) in $(0,\tau)\times Q_\varepsilon^\ell(\omega)$. The scaling of the viscosity is typical for the flow of Stokesian fluid through porous media [3,8]. The assumptions on the moduli $\mathbf{a}^\varepsilon,\boldsymbol{\pi}^\varepsilon$ and $\boldsymbol{\epsilon}^{s\varepsilon}$ are similar to those specified in [10] for microperiodic piezocomposites. In our case it suffices to extend conditions (A_1) and (A_2) given in [8] for elastic solid phase.

The conditions on the interface solid-fluid $\Gamma^\varepsilon(\omega)$ are specified by the following relations and hold for $t\in(0,\tau)$:

$$\begin{aligned}
&[\![\boldsymbol{\sigma}^\varepsilon\mathbf{n}]\!]=\mathbf{0},\quad [\![\Phi^\varepsilon]\!]=0,\quad [\![\mathbf{D}^\varepsilon\mathbf{n}]\!]=\zeta^\varepsilon,\\
&\mathbf{v}^\varepsilon=\dot{\mathbf{u}}^\varepsilon,\quad \mathbf{J}^{(+)\varepsilon}\cdot\mathbf{n}=0,\quad \mathbf{J}^{(-)\varepsilon}\cdot\mathbf{n}=0,
\end{aligned}\tag{14}$$

where

$$\boldsymbol{\sigma}^\varepsilon=\begin{cases}\mathbf{a}^\varepsilon\mathbf{e}(\mathbf{u}^\varepsilon(t,\mathbf{x},\omega))-\boldsymbol{\pi}^{\varepsilon(*)}\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon(t,\mathbf{x},\omega))\\ \qquad\qquad \text{in}\quad (0,\tau)\times Q_\varepsilon^s(\omega),\\ -p^\varepsilon(t,\mathbf{x},\omega)\mathbf{I}+\varepsilon^2\eta\mathbf{e}(\mathbf{v}^\varepsilon(t,\mathbf{x},\omega))\\ \qquad\qquad \text{in}\quad (0,\tau)\times Q_\varepsilon^\ell(\omega);\end{cases}\tag{15}$$

$$\mathbf{D}^\varepsilon=\begin{cases}\boldsymbol{\pi}^\varepsilon\mathbf{e}(\mathbf{u}^\varepsilon(t,\mathbf{x},\omega))+\boldsymbol{\epsilon}^{s\varepsilon}\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon(t,\mathbf{x},\omega))\\ \qquad\qquad \text{in}\quad (0,\tau)\times Q_\varepsilon^s(\omega),\\ \boldsymbol{\epsilon}^{\ell\varepsilon}\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon(t,\mathbf{x},\omega))\\ \qquad\qquad \text{in}\quad (0,\tau)\times Q_\varepsilon^\ell(\omega);\end{cases}\tag{16}$$

$$\begin{aligned}\mathbf{J}^{(\pm)\varepsilon}&=\mathbf{b}^{(\pm)\varepsilon}q^{(\pm)\varepsilon}\mathbf{E}_{\mathbf{x}}(\Phi^\varepsilon)+q^{(\pm)\varepsilon}\mathbf{v}^\varepsilon\\ &\quad-\mathbf{d}^{(\pm)}\nabla_{\mathbf{x}}q^{(\pm)\varepsilon}\qquad \text{in}\quad (0,\tau)\times Q_\varepsilon^\ell(\omega).\end{aligned}\tag{17}$$

Here $\mathbf{I}=(\delta_{ij})$ denotes the identity matrix. The interface potential (ζ-potential) ζ^ε may by assumed to be constant. We consider a more general case where $\zeta^\varepsilon=\zeta(\mathbf{x},T(\varepsilon^{-1}\mathbf{x})\omega)$. Since we are interested in the macroscopic equations, we do not consider boundary conditions on $\partial Q_\varepsilon(\omega)$. For the sake of simplicity we assume homogeneous initial conditions for $\mathbf{u}^\varepsilon,\mathbf{v}^\varepsilon,\Phi^\varepsilon$ and $q^{(\pm)^\varepsilon}$.

4. Stochastic homogenisation and macroscopic relations

Letting ε tend to zero in the sense of stochastic two-scale convergence in the mean we arrive at the homogenized equations. Without the assumptions of ergodicity the fields involved still depend on $\omega\in\Omega$.

Let $F\subset\Omega$ and $G=\Omega\setminus F$; F is assumed to be T-open and T-connected [1,7]. We observe that F plays the role of voids in local problems in the case of periodic microstructure. We set $\Psi=\mu(F)$, $Q_\tau=(0,\tau)\times Q$.

Selected results

(i) T is not necessarily ergodic:

Under physically plausible assumptions

$$\{\mathbf{u}^\varepsilon,\nabla_{\mathbf{x}}\mathbf{u}^\varepsilon,\Phi^\varepsilon,\nabla_{\mathbf{x}}\Phi^\varepsilon,\mathbf{v}^\varepsilon\}$$

stochastically two-scale converges in the mean to

$$\left(\chi_{\Omega\setminus F}\mathbf{u},\ \chi_{\Omega\setminus F}(\boldsymbol{\xi}+\nabla_{\mathbf{x}}\mathbf{u}),\ \Phi,\ (\boldsymbol{\theta}+\nabla_{\mathbf{x}}\Phi),\ \chi_F\mathbf{v}\right)$$

and, for instance

$$\mathbf{u}\in H^1(Q,I^2(\Omega))^n,\ \boldsymbol{\xi}\in L^2(Q,M^2(\Omega))^{n^2}.$$

Here χ_A denotes the characteristic function of set A.

The Darcy-Wiedemann law is nonlocal in time:

$$\begin{aligned}&\tilde{E}[\chi_F(\omega)(\mathbf{v}-\dot{\mathbf{u}})(t,\mathbf{x},\omega)]\\ &=\frac{1}{\rho^\ell}\int_0^t\mathbf{A}(t-s,\omega)(\mathbf{f}^g-\nabla_{\mathbf{x}}p-q\nabla_{\mathbf{x}}\Phi-\kappa\nabla_{\mathbf{x}}q)(s,\mathbf{x})ds\end{aligned}$$

where $\mathbf{f}^g$ depends on $(s,\mathbf{x})$ whilst q and Φ on $(s,\mathbf{x},\omega)$; $q=q^{(+)}+q^{(-)}$. The permeability matrix $\mathbf{A}=(A_{ij})$ is defined by

$$A_{ij}=E[\chi_F(\omega)\dot{\mathbf{w}}^{(i)}(t,\omega)\cdot\mathbf{e}_j],\quad i,j=1,2,3.$$

Here $\mathbf{e}_j$ stands for the j^{th} standard basis vector of $I\!R^3$. The matrix $\mathbf{A}$ is symmetric and positive definite 7,8]. The function $\mathbf{w}^{(i)}$ is a solution to the *flow cell problem*, given by Eqs. (4.14) in [7] cf. also [8].

(ii) T is ergodic on Ω:

The macroscopic fields $\mathbf{u},p,\Phi$ and q do not depend on ω.

The Darcy-Wiedemann law takes the form

$$\begin{aligned}&\langle\chi_F(\omega)(\mathbf{v}(t,\mathbf{x},\omega)-\dot{\mathbf{u}})(t,\mathbf{x})]\rangle\\ &=\frac{1}{\rho^\ell}\int_0^t\mathbf{A}(t-s)(\mathbf{f}^g-\nabla_{\mathbf{x}}p-q\nabla_{\mathbf{x}}\Phi-\kappa\nabla_{\mathbf{x}}q)(s,\mathbf{x})ds\end{aligned}$$

where $A_{ij}=\langle\chi_F(\omega)\dot{\mathbf{w}}^{(i)}(t,\omega)\cdot\mathbf{e}_j]\rangle$.

The macroscopic moduli $\mathbf{a}^h(\mathbf{x})$, etc., can be found by solving cell local problems, being stochastic counterpart of the local problems formulated in [3] for the periodic case. For instance, we have

$$\begin{aligned}a_{ijpq}^h(\mathbf{x})=\langle\chi_{\Omega\setminus F}(\omega)[a_{ijpq}+a_{ijmn}e_{mn}^\omega(\mathbf{B}^{(pq)})\\ -\pi_{kij}E_k^\omega(R^{(pq)})](\mathbf{x},\omega)\rangle\end{aligned}$$

The macroscopic stress tensor $\langle\boldsymbol{\sigma}^{(0)}\rangle(t,\mathbf{x}),\ \mathbf{x}\in Q$ is expressed by

$$\begin{aligned}\langle\boldsymbol{\sigma}^{(0)}\rangle(t,\mathbf{x})=\langle\chi_{\Omega\setminus F}(\omega)\boldsymbol{\sigma}^{s(0)}(t,\mathbf{x},\omega)\rangle\\ +\langle\chi_F(\omega)\boldsymbol{\sigma}^{\ell(0)}(t,\mathbf{x},\omega)\rangle.\end{aligned}$$

Explicit formula for $\langle\boldsymbol{\sigma}^{(0)}\rangle(t,\mathbf{x})$ generalizes that given in [4].

The stationary Darcy-Wiedemann law is obtained by letting t tend to infinity, cf. [11].

5. Final remarks

For other models of flow of electrolytes through porous media the reader is referred to [2,12,13]. Taking into consideration FCD (fixed charge density) one has to impose additional condition on the interface $\Gamma^\varepsilon(\omega)$ and the electroneutrality condition. A challenging problem is to use homogenisation methods for the case of finitely deformable skeleton, even hyperelastic. The permeability would then necessarily depend on strains. Such a dependence (nonlinear) is important even for small strain [14]. It is also important to include ion channels [15].

Appendix

Deterministic and random porous medium

Consider a fluid flow between two plane parallel surfaces, in x direction, under the pressure gradient dp/dx. The surfaces are located at $y = 0$ and $y = L$, respectively, and the channel extends in x and z directions infinitely. The velocity of flow varies with y and does not depend on x and z. It is known, that the mean velocity of fluid (averaged over the width L) is $\bar{v} = AL^2$ where $A = -(12\eta^\varepsilon)^{-1}dp/dx$ and $\eta^\varepsilon = \varepsilon^2\eta$. The total flow of fluid along the region enclosed between the planes $y = 0$ and $y = L$ on a unit height of z axis is $Q_0 = AL^3$.

If the channel is divided into $2n$ identical subchannels by the walls with vanishing thickness at planes $y = kL/2n, k = 1, 2, \cdots, 2n-1$, the total output of all system is $Q_0/(2n)^2$ (the width of each subchannel is $L/2n$). If half of channels is closed in such a way that only every second channel is opened, the total output of system is $Q_{2n} = Q_0/(8n^2)$. If $n = 1$, $Q_1 = Q_0/8$, if $n = 2$, $Q_2 = Q_0/32$, if $n = 3$, $Q_3 = Q_0/72$.

Let us now consider a stochastic counterpart of the system defined by the following conditions: (1) each subchannel can be opened or close with the probability 1/2; and (2) if two open channels are adjacent, then the separation wall between them vanishes and they create one channel with double width. If k open channels drop together, a new channel with the width $kL/2n$ arises.

If a channel is divided into two subchannels, each of width $L/2$, then the 4 situations are possible: open-open, open-closed, closed-open, closed-closed. The output is $Q_1^s = Q_0(1 + 1/8 + 1/8 + 0)/2^2 = 0.3125\,Q_0$. If $n = 2$, $Q_2^s = Q_0\,170/(64 \cdot 2^4) = 0.166\,Q_0$, and if $n = 3$, $Q_3^s = Q_0\,1412/(216 \cdot 2^6) = 0.102\,Q_0$. Because of a nonlinear dependence between output of channel and its width, the random system has a larger efficiency than the deterministic one with the same cross-section of open channels.

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