Roughness effects at microscale – reassessing Nikuradse’s experiments on liquid flow in rough tubes

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Abstract. The topic of incompressible fluid flow in rough channels is of practical interest in many diverse applications. It also forms the basis of our understanding of fluid-wall interactions, turbulent eddy generation, and their effect on the frictional pressure losses. Although this topic is also of fundamental interest, the work in this area is entirely guided by the experimental work of earlier investigators [1–6]. The works by Nikuradse [4] and Colebrook [5] constitute a major milestone from which useful empirical models are derived. As we approach the microscale, Nikuradse’s experimental work again is brought to focus, perhaps this time to gain an insight into the mechanisms affecting fluid-wall interaction in rough channels. In this paper, Nikuradse’s work is revisited in light of the recent experimental work on roughness effects in microscale flow geometries.

Key words: roughness, liquid flow, rough tube, Nikuradse’s experiments.

1. Introduction

Incompressible laminar fluid flow in ducts is the main focus of this paper. Theoretical solutions are available in the literature for laminar internal flow, starting with the Navier-Stokes equations which are based on the continuum model. In these classical works, the channel walls are considered to be smooth, with no slip condition at the wall. The issue of wall roughness is addressed through relative roughness, which is the ratio of one of the surface roughness parameters to the channel hydraulic diameter. It may be of historical interest to note that the term relative roughness was first proposed by Mises [7]. The surface roughness parameter is generally the equivalent sand grain roughness or average roughness $R_a$ obtained from a linear traverse of a profilometer on the channel wall in the flow direction. For the case of non-uniform roughness, the equivalent sand grain roughness concept is introduced to yield the same resulting pressure drop. A recent work by van Rij et al. [8] provides a more exhaustive treatment of this aspect.

Exhaustive experiments conducted by Nikuradse [4] with flow of water in smooth and rough macroscale pipes form much of our basis for understanding in this field. His results showed that the roughness did not affect the friction factor in the laminar region. Roughness also did not influence the laminar-to-turbulent transitions in pipes of different diameters.

The recent focus on microscale fluid flow, as seen by extensive survey papers published in the literature [9–21], has once again brought the focus to this topic. The experiments conducted since the late 80’s and in the 90’s focusing on microscale and miniscale channels, ranging from 10 $\mu$m to about 1 mm, showed a departure from the conventional laminar theory for rough pipes. The following questions were then raised in providing a possible explanation for this departure in laminar flow characteristics of a liquid.

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2. Reassessment of Nikuradse’s experimental data

Nikuradse [4] conducted experiments with water in circular pipes to study the laminar and turbulent regions, covering Re from 600 to $10^6$. The inner diameters of the pipes were 25, 50 and 100 mm. To create the roughness on the inside walls of the pipes, he first sifted sand grains so that the grains were of uniform size. He then used Japanese lacquer to stick the sand grains to the walls of the pipe. The pipes were first filled with Japanese lacquer and then emptied, leaving a thin tacky film on the wall. The pipe was then filled with sifted sand grains of desired diameters and emptied. The pipes were then dried for 3–4 weeks. To assure good adherence of sand particles to the wall, the pipe was once again filled with Japanese lacquer and emptied. This resulted in a thin coating of lacquer which improved sand adhesion to the wall. The height of the sand particles was measured by repeating the same procedure on a flat glass plate and using a height gauge. The resulting relative roughness values ranged between 0 and 3.3%. The exact profile of the surface, covered with sand grains and a film of Japanese lacquer, was however not obtained.

In the friction factor evaluation, pipe diameter has a very large influence since the pressure drop along a given pipe length varies as $1/D^5$ for a given fluid flow rate. In Nikuradse’s experiments, the diameter of the pipe was calculated from the volume of water required to fill the pipe and the length of the pipe. This measurement method provided a mean diameter along the pipe length without conducting a destructive testing. The measurement uncertainty however could not be determined as the amount of water that was left as a film was difficult to estimate. Assuming the water film thickness to be on the order of 5 $\mu$m, the maximum uncertainty is estimated to be 10/25000, or 0.04% in the smallest tube tested. It should be noted that the pipe diameter used in his work was the base diameter of the pipe before the lacquer or the sand grains were applied.

The pressure drop was measured at several intermediate locations using pressure probes of 2 mm outside diameter that extended to the centre of the pipe. The micromanometers were used for pressure drop measurement. The uncertainty associated with the pressure drop measurement in the lower range of turbulent flow experiments is estimated to be on the order of ±0.5 mm manometer height of water column in a 10 mm column, or 5%. This resulted from the human error in reading the manometer.

The laminar region of the experiments conducted by Nikuradse was toward the lower Re range of his experiments, which were conducted with Re as high as $10^6$. Although this is toward the low end of the range, the flow rate measurement does not seem to be influenced by his measurement techniques. He used an overflow tank to measure the volume of water collected over a given time. The volume measurement method involves only the human error in setting the clock, along with the accuracy of the weighing scale. It is estimated that the uncertainty related to the flow measurement is quite low, approximately below 1%. The turbulent transition was also investigated through direct flow visualization. Further, the friction factors for smooth and different roughness tubes in the laminar region yielded pressure gradients that were within less than 3% of each other, and agreed with the theoretical laminar predictions.

Nikuradse carefully established the entrance region effects and took his measurements in the fully developed region. He also accounted for the presence of the pressure drop probes, which were placed in the centre of the pipes to avoid any wall effects.

The errors associated with the pressure drop measurement in Nikuradse’s experiments in the laminar flow region are however significantly higher. For a Reynolds number of 600, the pressure drop corresponds to a water column height of only about 0.1 mm/m pipe length. For a measurement section length of 1.5 m between two probe stations, this still amounts to be considerably smaller than the estimated uncertainty of ±0.5 mm of water head.

The overall uncertainty associated with Nikuradse’s experiments to measure pressure drop is estimated to be between 3 to 5% in the turbulent region, but it is expected to be significantly higher in the laminar region.

Nikuradse’s results in the laminar region agreed with the earlier results of Schiller [3], who conducted experiments directed at finding the critical Reynolds numbers at the laminar-turbulent transition. Schiller’s pipe diameters ranged from 8 to 21 mm. He reported that the transition Reynolds number was independent of the surface roughness. However, the uncertainty in Schiller’s data needs to be carefully evaluated further.

In literature, there are no reports contradicting the effect of roughness on friction factor in the laminar range for conventional diameter tubes. It may be tempting to conclude that laminar flow at macroscale (above 3 mm), and perhaps well into miniscale (below 3 mm), exhibit no effect of wall roughness on the friction factors. However, in light of the questions raised here regarding the uncertainty in pressure drop measurements in Nikuradse’s data, there is a need to re-evaluate the roughness effects through carefully controlled experiments in the laminar region for large diameter tubes as well.

3. Recent experimental observations on roughness effect at microscale

3.1. Effect of roughness on friction factor. The channel classification scheme proposed by Kandlikar and Grande [18] is used in this paper. According to this classification, channels in the range 10 to 200 $\mu$m are classified as microchannels, and between 200 $\mu$m to 3 mm are classified as minichannels.

Some of early experimental works in this area did not account for the inlet and exit losses and the entrance region effects. For example, Pfahler et al. [10] conducted experiments with very small diameter microchannels, with the channel heights varying from 0.5 to 4.65 $\mu$m, channel width varying from 95 to 115 $\mu$m, and relative roughness of around 1%. The results show a large degree of scatter, and in general a lower friction factor as compared to the theoretical predictions. The uncertainty in the channel size and shape may be responsible
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for the large variation, and additional confirmation is needed before any conclusions can be drawn from this study. Similarly, the experimental data of Peng et al. [29] showed very early transition to turbulent flow for smooth rectangular channels with hydraulic diameters in the range from 133 to 367 µm. The relative roughness of their channels was between 0.6 and 1%. The uncertainties in diameter measurement were not stated and are suspected to affect the results that might be misinterpreted as early transition to turbulence based on the pressure drop measurements. Nevertheless, this work marks the beginning of our attempts to unravel the roughness effects at microscale. Later investigators worked on refining the experimental uncertainties, accounted for entrance and exit losses, and considered the pressure drop in the developing region.

A number of researchers [28–40] explored the effect of roughness on fluid flow at microscale. One of the convincing studies showing the roughness effect on laminar flow was conducted by Mala and Li [31] using circular microchannels with diameters ranging from 50 to 254 µm and surface relative roughness of up to 3.5%. They eliminated the entrance and exit losses and the entrance region pressure drop by considering two samples of the same tube of different lengths. Their results clearly indicate that as the tube diameter decreases, (the relative roughness increases for the same roughness surface), the friction factor increases for the same Reynolds number. The same value of surface roughness results in a higher relative roughness in smaller diameter tubes, and a higher friction factor. The authors proposed a roughness-viscosity model to account for the surface roughness effects.

Pfund et al. [32] conducted experiments with tap water flowing in high aspect ratio microchannels with depths varying from 128 to 521 µm, and Re varying from 60 to 3450. The effect of roughness was investigated by introducing a rough bottom plate. The pressure drop was measured in the channel itself. Their results indicate a strong possibility that the roughness causes an increase in laminar flow friction factors. The authors pointed out the large uncertainties in their experiments and suggested further studies to confirm their findings.

Guo and Li [17] present a good survey of earlier investigators’ and their own experimental work showing the roughness effects. Their experimental data for rough microchannels with diameters varying from 128 to 179 µm showed that the friction factors increased with an increase in the relative roughness (in the range between 2 and 4.3%). They also presented an interesting discussion on the effects of the wall frictional forces on the bulk flow, indicating that these effects are expected to be responsible for early transition to turbulent flow and increased friction factor and Nusselt number.

Qu et al. [33] conducted experiments in silicon microchannels and identified roughness as an important factor in the development of microfluidic devices in biological applications.

Kandlikar et al. [34] systematically varied the roughness of stainless steel tubes using acid etching and studied the effect of roughness on pressure drop and heat transfer. The study focused on minichannels with two tubes of 1.032 and 0.62 mm inner diameters. The relative roughness varied from 0.16 to 0.36% and the tests were conducted with water over a Re range from 500–2600 for 1.032 mm tube and 900–3000 for 0.62 mm tube. Figure 1 shows the roughness effect in the smaller diameter tube, with a higher pressure drop for tubes with higher relative roughness values. The transition Reynolds number was also affected. The larger diameter tube did not exhibit any roughness dependency over the experimental data range. They also recommended systematic studies on roughness effect for microchannels, where the roughness effects were expected to be higher.

Tu and Hrnjak [28] conducted experiments with R-134a in five rectangular microchannels with hydraulic diameters varying from 69.5 to 304.7 µm and aspect ratios between 0.09 and 0.24. The Reynolds numbers were varied from 112 to 9180. Their results for smooth channels showed that the conventional theory for friction factor was applicable to all channels over the entire laminar region, and the critical Reynolds number was also unaffected. For one channel with the highest relative roughness (0.35%), the friction factors were 9% higher than the theoretical predictions, and the critical Reynolds number was 1570.

Wu and Cheng [35] conducted a systematic study on 13 different trapezoidal silicon microchannels with two different nominal channel heights of 56.22 and 110 µm. The relative roughness of the surface varied from 0.00326 to 1.09%. They plotted the apparent friction factor based on the pressure measurements in the inlet and the outlet manifolds. The entrance and exit losses and the developing region loss were included in the apparent friction factor. Nevertheless, their data clearly shows the influence of the wall roughness on friction factor. Figure 2 shows the pressure drop for two channels with different relative roughness values as indicated in the figure. The lower pair of plots is for triangular sections with the width of the base around 110 µm and channel height of around 110 µm. The friction factors for the rough channel are seen to be around 20% higher than those for the lower roughness channels. Similar observations can be made for the upper pair, which represents two channels with the channel height around 110 µm, and the two parallel sides of the trapezoid around 270 and 430 µm respectively.
Celata et al. [36] verified that the conventional theory holds good for microchannels down to 30 µm diameter. The experiments with roughened channels, with the diameter varying from 70 to 326 µm, indicated that the roughness effect was most prominent with the smallest diameter tubes. The relative roughness varied from 0.07 to 0.62%. The presence of viscous heating was also demonstrated, but the heating is expected to lower the viscosity and reduce the friction factor, whereas the experimental data yielded a friction factor that was higher than predicted by the continuum theory. Figure 3 shows their results comparing friction factors for roughened glass tubes of 126 and 299 µm inner diameters. Assuming the surface roughness values to be nearly identical, the relative roughness for the 126 µm diameter tube will be higher than the 299 µm diameter tube. The resulting friction factor for the smaller diameter tube is seen to be higher over the entire laminar range.

The effect of surface roughness on flow results from: i) flow obstruction, and ii) flow constriction. The effect of structured roughness elements on the flow has been studied in the literature. For example, Webb [40] studied the flow separation during fluid flow over repeating roughness elements. Flow separated from the wall following the roughness elements and then reattached at a distance 6–8 times the height of the element. For surface roughness resulting from a manufacturing process, such as machining or surface treatment, the roughness elements are similar in nature and are closely spaced. The flow encounters the leading element and skims over a pocket of circulating fluid before passing over the next element. Kandlikar et al. [38] proposed a constricted flow model, in which the flow cross-sectional area was considered to be equal to the constricted flow area (or free-flow area in compact heat exchanger terminology). For the case of the saw-tooth roughness discussed earlier, the results of Fig. 4 are replotted using the constricted flow diameter in Reynolds number and friction calculations. Figure 5 shows the results based on the constricted flow diameter. It can be seen that the results for the rough tubes match with the laminar flow predictions based on the same constricted flow parameter.
3.2. Modelling of roughness structures. Modelling roughness effects on microscale friction factor is important in designing microscale fluidic devices. Several researchers, e.g. Bahrami et al. [37], Kandlikar et al. [38] and Taylor et al. [41] present some modelling concepts in this area. Bahrami et al. [37] assumed the wall roughness to possess a Gaussian distribution. For isotropic roughness, they developed expressions for frictional resistance based on the standard deviation in the roughness profile and derived a correction factor to account for the wall roughness. Their work was shown to correlate the available data from literature well.

Kandlikar et al. [38] and Taylor et al. [41] proposed the average roughness as the sum of \( F_\text{p} \) and \( R_p \), where \( F_\text{p} \) is the distance between the main profile mean line and the main profile mean line and \( R_p \) is the distance between the main profile mean line to the average height of the peaks. Using this concept, the definitions of various roughness parameters is depicted in Fig. 6. These parameters were used in representing the saw-tooth profile roughness structures. As shown in Fig. 5 in the earlier section, the model was successful over the range of experiments conducted by Kandlikar et al. [38] up to a maximum relative roughness of around 15%.

3.3. Transition to turbulent flow due to roughness at microscale. Turbulent flow transition is another topic that is being pursued in evaluating the roughness effects on microscale fluid flows. A number of researchers, including Idelchik [42], Peng et al. [30], Celata et al. [36], Mala and Li [31], Kandlikar et al. [38], Schmitt and Kandlikar [39] have experimentally confirmed the effect of roughness on transition to turbulence. The following equation is proposed by Kandlikar et al. [43] based on their experimental data on saw-tooth roughness structures in rectangular minichannels (Kandlikar et al. [38] and Schmitt and Kandlikar [39]).

Laminar-to-turbulent transition equations:

\[
0 < \varepsilon / D_{h,cf} \leq 0.08 \quad \text{Re}_{t,cf} = 2300 - 18,750 \left( \varepsilon / D_{h,cf} \right)
\]

\[
0.08 < \varepsilon / D_{h,cf} \leq 0.15 \quad \text{Re}_{t,cf} = 800 - 3270 \left( \varepsilon / D_{h,cf} - 0.08 \right)
\]

4. Numerical modelling of roughness structures in laminar flow

A number of investigators have studied microscale roughness effects in microchannel flows using numerical simulation. Hu et al. [44] numerically investigated the rectangular prism roughness elements. The Reynolds number range investigated was from 0.001 to 10, and the channel height varied from 5 to 50 \( \mu \)m. The three-dimensional analysis showed the effect of surface roughness in terms of the element height, and size relative to the channel height. The effect of the roughness elements was presented in terms of channel height reduction. This expression was obtained from their numerical data. The height and spacing of the roughness are shown to affect the flow in a complex way in different regions. Kleinstreuer and Koo [45] used a porous layer model in representing the roughness elements in the relative roughness range varying from 0.5 to 2%.

Rawool et al. [46] present the results of the numerical simulation obtained by systematically varying the height and pitch of the roughness ridges in a rectangular microchannel. These results are very illustrative in depicting the effect of surface element size and shape on the fluid flow. Further experimental validation of these results is needed before the results can be employed in practical design of microscale devices.

5. Conclusions

It is very interesting to see the progression of research in developing our understanding of fluid flow phenomena at microscale. The researchers in the late 80s and 90s struggled with obtaining accurate experimental data and recognizing the importance of the experimental uncertainties associated with measurements of channel dimensions and flow parameters. As a result of these consorted efforts, the continuum theory has been shown to be valid for liquid flows in smooth microchannels.

From the literature survey, we can conclude that surface roughness plays an important role in fluid flow in microchannels and minichannels. For macrochannels, the experiments of Schiller [3] and Nikuradse [4] indicate that roughness has no effect on the laminar flow friction factor in rough tubes with relative roughness of \( 0 < \varepsilon / D \leq 0.05 \). The transition to turbulence is also seen to be unaffected by the presence of roughness structures in the large diameter tubes investigated by these
researchers. The shape of the roughness structure, ribs or uniform surface roughness, did not make any difference either. However, since these experiments were conducted at the lower end of their pressure drop measurement instrument range, large uncertainties are suspected to be present in the pressure drop measurements in the laminar region. Further verification of their laminar flow results with more accurate instrumentation in macrochannels is therefore warranted.

For microchannels, as well as for minichannels in its lower dimensional range, it can be concluded that roughness plays a role in the friction factor as well as in the transition to turbulence. The experimental confirmation from recent literature is quite convincing to this effect. The role of roughness and the mechanisms affecting this laminar flow behaviour are not yet clear. Further study on this topic is expected to be pursued aggressively by researchers throughout the world.

REFERENCES

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