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Unfalsified control of manipulators: simulation analysis

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Abstract. In this paper we present results of systematic and comprehensive simulation analysis of the Tsao & Safonov unfalsified controller for complex robot manipulators. In particular, we show that the controller falsification procedure yields the closed-loop unfalsified controller, which accomplishes the control objective, within a finite and relatively short time interval with the number of invocations of linear programming based unfalsified controller selection procedure being relatively small. We also present some conclusions resulting from the investigation of the effect of such elements as manipulator structure complexity, prior knowledge about disturbances, reference trajectory and assigned closed-loop spectrum on unfalsified controller performance and computational complexity.

Key words: unfalsified control, manipulator, linear programming.

1. Introduction

In the control of manipulators which parameters and dynamics are partially known of the commonly accepted solution are adaptive control algorithms [1–5]. These algorithms were constructed according to the certainty equivalence principle, the adaptive laws had been obtained by means of Lyapunov design techniques. Although these results have been known for some time they cannot be considered as completely satisfactory. Better algorithms are expected to be obtained by deeper understanding of the existing paradigms as well as by exploring alternative approaches to robot control. The second option refers to such approaches as unfalsified control, [6, 7].

In [7] the authors undertake the issue of taking advantage of *a priori* mathematical knowledge about the plant and disturbances in the unfalsified control strategy, which is essentially empirical. The considerations refer to a specific plant which is a horizontal 2R robotic manipulator. In the equation of motion bounded disturbances were included with the model parameters unknown. The authors constructed a new robot manipulator controller by integrating the principles of the unfalsified control design strategy and model based techniques, which can be perceived as a robust adaptive computed torque control algorithm.

The literature reports many robust adaptive control algorithms for manipulators. Very profound surveys can be found in [2, 3]. Usually, the design consists of an adaptive controller that implements a feedforward term, which compensates the modelled dynamics, and of a sliding mode [8, 9] or an adaptive PD [10] control law that overcomes the unmodelled dynamics and noise. Sometimes additional and specific restrictions are imposed with respect to a prior knowledge of unknown parameters and unmodelled dynamics [8]. In the Tsao & Safonov control algorithm [7] the compensating feedback linearization term is present but the effect of unmodelled dynamics is surmounted by the unconventional "adaptive law". This law is based on linear programming methods and significantly differs from typical adaptive laws used in manipulators robust controllers.

The Tsao & Safonov controller belongs to the class of supervisory switching controllers. Currently active feedback controller is switched off and replaced whenever its ability to meet the performance goals is falsified by evolving experimental data. Thus common in robotics Lyapunov stability analysis methods cannot be applied here. Unfortunately the arguments motivating internal stability of the closed-loop system and accomplishing the control objective, provided in [7], are not fully formal. On the other hand, the simulation results presented in [7] reveal a number of interesting properties of the unfalsified controller. In particular, it can very well cope with bounded uncertainties. Moreover, the unknown model parameters are adjusted rapidly and very precisely as compared with common adaptation methods having continuous adaptation rules.

If these simulation results were valid for more complex manipulators they might be quite attractive for a robotican. Therefore it seems reasonable to carry out systematic and comprehensive simulation experiments that would verify and eventually extend the results from [7]. This thread has been undertaken in this paper. For this purpose a special simulation package in the Matlab environment has been developed with implemented spectrum

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of manipulator models, including an industrial one. Numerous experiments have been carried out on a large scale. Special attention was paid to: controller performance with respect to uncertainties, the number of switchings of controllers, distribution in time of the switching moments, computational complexity, model parameter adaptation upon uncertainties, the reference trajectory and the closed-loop spectrum.

The analysis of the simulation experiments results entitles to state that the results obtained for horizontal 2R robot manipulator are valid for more complex manipulators. However, the higher manipulator complexity, the lower admissible uncertainties for proper controller performance and the higher number of switching of controllers, the worse and less rapid parameter adjustment.

The paper is organized as follows. Section 2 reviews the Tsao & Safonov control strategy for manipulators and points out several open problems, which are undertaken further on. Section 3 briefly describes the software used in simulation experiments and provides some basic information about the experimental process. Section 4 presents conclusions obtained on the basis of experiments and illustrates them with selected simulation results. Finally, Section 5 sums up the paper.

2. Tsao & Safonov unfalsified controller – simulation problems formulation

We consider a manipulator with rigid links and nonflexible joints. It follows from the Euler-Lagrange equations that the dynamics of such manipulators can be described by the following equation:

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + G(q) + \zeta = u. \tag{1}$$

In (1) M stands for the positive definite manipulator inertia matrix, C represents centrifugal and Coriolis torques, G is a vector of gravitational torques, q, u denote vectors of generalized positions and generalized forces. ζ represents disturbances. It is assumed that $|\zeta_i| \le \bar{\zeta}_i$, where $\bar{\zeta}_i$ denotes the i-th component of ζ and $\bar{\zeta}_i$ is known.

The Eq. (1) can be rewritten in the linear regression model form,

$$Y(\ddot{q}, \dot{q}, q)\theta_* + \zeta = u. \tag{2}$$

where θ_* is a vector of the system parameters and the regression matrix Y is a known function of measurable signals q, $\dot{q}$, $\ddot{q}$.

Within the unfalsified control framework [7] the manipulator (1) is modeled by a set of trajectories P, where

$$\mathsf{P} \triangleq \{(\ddot{r}, \dot{r}, r, \ddot{q}, \ddot{q}, q, u) \mid M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + G(q) + \zeta = u, \quad r \in \mathcal{C}^2(\mathbb{R}, \mathbb{R}^n)\}. \tag{3}$$

The unfalsified control theory is a model free approach and therefore the equation is secondary in relation to the set of trajectories.

The unfalsified controller design strategy is based on the Unfalsified Control Theorem (UCT) [6] which requires a specification of three sets: $\mathcal{K}$ - the set of potential controllers for the plant, $\mathcal{T}_{\mathrm{spec}}$ – the set of desired trajectories of the interconnection plant - controller and $\mathsf{P}_{\mathsf{data}}$ – measurement information.

In the unfalsified control the idea is to interconnect the manipulator (3) with a controller from the set of candidate controllers $\mathcal{K}$ and keep this interconnection as long as this controller is unfalsified (in the meaning of the UCT). If at the time instant t the interconnected controller is falsified then another controller from the set of unfalsified controllers is connected to (3).

The set $\mathsf{P}_{\mathsf{data}}$ is defined as follows.

$$\mathsf{P}_{\mathsf{data}} \triangleq \{(\ddot{r}, \dot{r}, r, \ddot{q}, \dot{q}, q, u) \mid \mathsf{P}_\tau \begin{bmatrix} \ddot{q} \\ \dot{q} \\ q \end{bmatrix} = \begin{bmatrix} \ddot{q}_{\mathsf{data}} \\ \dot{q}_{\mathsf{data}} \\ q_{\mathsf{data}}, \end{bmatrix}, \quad \mathsf{P}_\tau u = u_{\mathsf{data}}, \quad r, q \in \mathcal{C}^2(\mathbb{R}, \mathbb{R}^n)\}, \tag{4}$$

where P_τ is a truncation operator[1].

The sets $\mathcal{K}$ and $\mathcal{T}_{\mathrm{spec}}$ are not mutually independent and they have to be constructed simultaneously at the certain stage of the controller design process. Notice that $\mathcal{K}$ has to contain at least one controller such that the interconnection of this controller and the manipulator meets the closed loop system specification characterized by $\mathcal{T}_{\mathrm{spec}}$.

The manipulator dynamical equation (1) need not, but it can be utilized to construct $\mathcal{K}$ and $\mathcal{T}_{\mathrm{spec}}$. We can go further and construct both sets $\mathcal{K}$ and $\mathcal{T}_{\mathrm{spec}}$ on the basis of the well established and commonly accepted control law. The authors of [7] took advantage of the computed torque control strategy,

$$u = M(q)(\ddot{q}_r + 2\lambda\dot{\tilde{q}} + \lambda^2\tilde{q}) + C(q, \dot{q})\dot{q} + G(q) \tag{5}$$

(here $\lambda > 0$, q_r denotes the reference trajectory and $\tilde{q} = q_r - q$) and of the closed-loop system (1,5) dynamics Eqs:

$$M(q)(\ddot{\tilde{q}} + 2\lambda\dot{\tilde{q}} + \lambda^2\tilde{q}) = \zeta. \tag{6}$$

$\mathcal{K}$ and $\mathcal{T}_{\mathrm{spec}}$ have been defined in [7], in the following way:

$$\mathcal{K} = \{\mathsf{K}(\theta) \mid \theta \in \mathbb{R}^m\}, \tag{7}$$

$$\mathsf{K}(\theta) = \{(r, \dot{r}, \ddot{r}, q, \dot{q}, \ddot{q}, u) \mid u = M(q, \theta)(\ddot{r} + 2\lambda(\dot{r} - \dot{q}) + \lambda^2(r - q)) + C(q, \dot{q}, \theta)\dot{q} + G(q, \theta)\}; \tag{8}$$

$$\mathcal{T}_{\mathrm{spec}} = \{\mathsf{T}_{\mathrm{spec}}(\theta) \mid \theta \in \mathbb{R}^m\}, \tag{9}$$

$$\mathsf{T}_{\mathrm{spec}}(\theta) = \{(r, \dot{r}, \ddot{r}, q, \dot{q}, \ddot{q}, u) \mid |M(q, \theta)((\ddot{r} - \ddot{q}) + 2\lambda(\dot{r} - \dot{q}) + \lambda^2(r - q))| \le \bar{\zeta}\}; \tag{10}$$

The set $\mathsf{K}(\theta)$ in (8) characterizes a candidate controller for the manipulator P. $\mathsf{T}_{\mathrm{spec}}(\theta)$ specifies the desired behaviour of the interconnection of P and $\mathsf{K}(\theta)$.

The controller falsification process is permanent, its mechanism has to be based on UCT. It follows from this

[1] $[P_\tau x](t) \triangleq \begin{cases} x(t) & \text{if } 0 \le t \le \tau \\ 0 & \text{otherwise} \end{cases}$

theorem that $\mathsf{K}(\theta)$ is unfalsified by measurement information P_{data} iff for each $(r, \dot{r}, \ddot{r}, q, \dot{q}, \ddot{q}, u) \subset \mathsf{P}_{\text{data}} \cap \mathsf{K}(\theta)$ there exists at least one quadruple $(\hat{q}, \dot{\hat{q}}, \ddot{\hat{q}}, \hat{u})$ such that $(r, \dot{r}, \ddot{r}, \hat{q}, \dot{\hat{q}}, \ddot{\hat{q}}, \hat{u}) \subset \mathsf{P}_{\text{data}} \cap \mathsf{K}(\theta) \cap \mathsf{T}_{\text{spec}}(\theta)$.

Direct implementation of UCT is practically impossible. Therefore alternative methods that would return unfalsified controllers in the sense of UCT have to be found. Considering the interconnection of the manipulator P and the controller $\mathsf{K}(\theta)$. It is not difficult to show that the behaviour of this interconnection is characterized by the following equation:

$$M(q,\theta)(\ddot{\tilde{q}} + 2\lambda\dot{\tilde{q}} + \lambda^2\tilde{q}) = u - Y(\ddot{q}, \dot{q}, q)\theta. \tag{11}$$

The Eq. (11) leads to three important observations.

- Let $\tilde{u} := u - Y(\ddot{q}, \dot{q}, q)\theta$. If
$$|\tilde{u}| \leq \bar{\zeta} \tag{12}$$
on the time interval $[0, t]$ then $\mathsf{K}(\theta)$ is unfalsified in the sense of UCT at the time instant t.
- The set $\mathcal{K}$ contains at least one controller, $\mathsf{K}(\theta_*)$, which is unfalsified on the whole time interval.
- If $\mathsf{K}(\theta)$ is an unfalsified controller on the whole time interval and $M(q,\theta) > \delta I$, $\delta > 0$, then $\tilde{q}, \dot{\tilde{q}}, \ddot{\tilde{q}} \in \mathcal{L}_\infty$ and the interconnection is internally stable.

In the context of falsification procedure the first observation is of fundamental significance. With the help of (12) the set of parameters representing the unfalsified models on the time interval $\mathbb{T}$, $G_{\mathbb{T}}$, can be constructed as follows:

$$G_{\mathbb{T}} = \{\theta \mid |u_i(\tau) - Y_i(\ddot{q}(\tau), \dot{q}(\tau), q(\tau))\theta| \leq \bar{\zeta}_i, \quad i = 1, \ldots, n,\ \tau \in \mathbb{T}\}. \tag{13}$$

In (13) u_i, Y_i, $\bar{\zeta}_i$ denote the i-th row of u, Y and ζ respectively. Usually $\mathbb{T} = [0, t]$. It follows that $\mathsf{K}(\theta)$ is unfalsified at the time instant t by the measurement information P_{data} iff $\theta \in G_{[0,t]}$.

Thus, if at the time instant t the controller $\mathsf{K}(\theta)$ interconnected to the manipulator P appears to be falsified then at the time instant $t+$ (the time instant just after t) a new controller $\mathsf{K}(\theta')$ has to be interconnected to P, such that $\theta' \in G_{[0,t]}$. In the light of the second observation this procedure is well defined because $G_{[0,t]}$ is nonempty for all t.

It has been recognized in [7] that if the interval $\mathbb{T}$ is discretized then $G_{\mathbb{T}}$ is a convex polytope and, moreover, computation of an element of a convex polytope is a linear programming problem for which there are many good computational algorithms. Hence it is proposed in [7] to update the parameters vector θ according to the following rule. If $\theta(t) \in G_{\mathbb{T}}$ then $\theta(t+) = \theta(t)$. Otherwise $\theta(t+)$ takes value of θ_c which is the center of the largest ball that fits inside the convex polytope $G_{\mathbb{T}}$:

$$\theta_c = \arg\max_{\theta \in G_{\mathbb{T}}} \operatorname{dist}(\theta, \partial G_{\mathbb{T}}), \tag{14}$$

where $\partial G_{\mathbb{T}}$ denotes the boundary of the set $G_{\mathbb{T}}$. θ_c can be computed by solving the following linear programming problem, [7]:

$$\theta_c = \arg\max_{\theta \in G_{\mathbb{T}}} \delta \tag{15}$$

subject to

$$\begin{aligned} &\delta \geq 0 \\ &Y_i(\ddot{q}(\tau), \dot{q}(\tau), q(\tau))\theta + (-u_i(\tau) + \bar{\zeta}_i) \\ &\quad -\delta\|Y_i(\ddot{q}(\tau), \dot{q}(\tau), q(\tau))\| \geq 0 \\ &-Y_i(\ddot{q}(\tau), \dot{q}(\tau), q(\tau))\theta + (u_i(\tau) + \bar{\zeta}_i) \\ &\quad -\delta\|Y_i(\ddot{q}(\tau), \dot{q}(\tau), q(\tau))\| \geq 0 \end{aligned} \tag{16}$$

$\forall i$ and $\forall \tau \in \mathbb{T}$.

The control law in (8) in conjunction with the controller falsification procedure depicted above constitutes the Tsao & Safonov ufalsified control strategy.

This strategy can be perceived as a robustified adaptive computed torque control algorithm. Careful analysis of the design procedure of this control algorithm leads to a number of remarks.

The Tsao & Safonov control strategy does not guarantee that the unfalsified controller on the whole time interval $[0, +\infty)$ will be derived within the finite time period $[0, t^*]$, $t^* < +\infty$. Since $G_{[0,t]}$ has to be discretized in time we can be sure that the manipulator is interconnected with unfalsified controller along a certain sequence $\{t_k+\}_{k=0}^{+\infty}$, which does not necessarily mean that on the whole time interval $[t_k, t_{k+1}]$ the connected controller is unfalsified. At this moment we assume that if θ belongs to the discretized in time set $G_{[0,t]}$ then it also belongs to the original $G_{[0,t]}$. Therefore the stability of the closed-loop system and the quality of the controller performance cannot be deduced directly from (11).

The computational complexity of the parameters θ update algorithm is proportional to the number of past experiments. The complexity could be reduced by enlarging the time probing period. However, this might badly affect the internal stability and the controller performance as there seems to be a conflict between internal stability and computational complexity.

On the other hand if the closed-loop system can be initialized such that the unfalsified controller is derived within a finite time interval then it follows directly from (11) that the closed-loop system is internally stable and the control objective is achieved.

To the best knowledge of the authors the formal solutions to the problems depicted above have not been found yet. However, the results of simulation experiments carried out on the horizontal double pendulum type manipulator, reported in [7], suggest that the number of switching the falsified controllers off is practically finite. Notice that this result was obtained for a simple plant. Therefore the fundamental question is whether it is valid for manipulators of a more complex structure like EDDA, IRp-6, Stäubli RX-90 or SCARA. If the answer appears to be positive the important information will be the total number of switchings and the time instant of the last controller switching. These parameters will be denoted by κ and t_s respectively.

Moreover, several other properties that are attractive for roboticans have been shown in [7]. In particular, the unfalsified controller can cope with bounded disturbances quite well. The controller can be obtained rapidly and the difference between the parameters vector θ representing the unfalsified controller and the true system parameters θ_* can be very small. Recall that in consequence the control law is very close to the control law that would be obtained if the true manipulator parameters θ_* were known. The natural question is how these properties transfer to the control systems with more complex manipulators.

There are also a few additional specific questions of secondary importance the answers to which could complement the results from [7].

Suppose that it is possible to derive the closed-loop unfalsified controller $\mathsf{K}(\theta)$ within a finite number of steps. How much θ differs from θ_*? Can we lower the difference by lowering $\bar{\zeta}$? Will the number of invocations of linear programming algorithm grow in that case? Notice that the smaller $\bar{\zeta}$ (i.e., the smaller disturbances), the better the worst case parameter estimates. Moreover, the better the parameter adjustment, the more data are necessary to achieve this, and therefore the value of t_s should grow.

Consider Eq. (6). Does the assigned dynamics of the closed loop system have any effect (regular effect) on the speed of searching for the unfalsified controller? Is it possible to provide certain tips for the designer of the controller, in particular how to choose the parameter λ? Observe that the larger the λ in (5) then, on the basis of (6), the faster the convergence of $\tilde{q}$, and the shorter the time necessary to obtain the sufficiently various set of constraints in (13), and the quicker the steadier unfalsified controller is derived.

Finally, does the frequency spectrum of the reference trajectory affect the convergence of θ ? Observe, that the more varying the reference trajectory, the more varying the set of constraints in (13) and the better parameters adjustment is expected.

3. Simulation experiment organization

Formal solution of the problems posed at the end of the Section 2 seems to be a nontrivial task. These problems, however, could be better understood and partially solved by carrying out systematic and comprehensive simulation experiments.

To fulfill the postulate of systematic experiments, a special software package for simulation – Sumnis[2] was written in the Matlab environment. The user interface of Sumnis is shown in Fig 1.

To ensure comprehensiveness of experiments the user of Sumnis is offered an access to the graphs of all of the essential variables in the control system and the possibility of setting values of all parameters of the particular components of the investigated dynamical system.

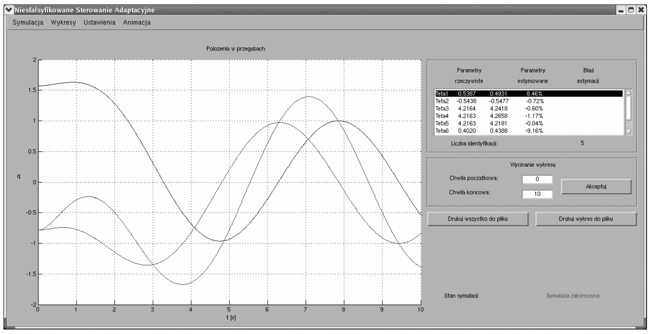


Fig. 1. Sumnis – the user interface

Special attention has been paid to a possibility of affecting the parameters of the controlled plant, the reference trajectory and the external disturbances. The user has the four manipulators to choose from: Edda [5], IRp-6 [13], Stäubli RX-90 [1] and SCARA. If necessary he can incorporate easily new models of manipulators to the system. The reference trajectory and the external disturbance can be any function that is possible to be defined in Matlab.

As in [7] actuators for each joint were taken into account. Denoting by u_i^c the output signal of the controller associated with the i-th joint, by u_i^s the output signal of the actuator driving the i-th joint the relationship between these variables and the i-th input of the manipulator can be expressed as follows: $\tau_i \dot{u}_i^s + u_i^s = u_i^c, u_i = u_i^s - \zeta_i$. The user can set any value for $\tau_i \geq 0$.

In the experiment five focal areas have been distinguished:

- controller performance: no disturbance case;
- controller performance: bounded disturbance case;
- disturbances prior information and computational complexity;
- closed-loop assigned spectrum and computational complexity;
- reference trajectory and accuracy of estimates.

The results of the first focal area should show the best possible controller performance and be a reference point for the results obtained in the further stage of the experiment. It is assumed that the measure of the controller performance is a resultant of tracking quality, internal stability and computational complexity. Evaluation of the tracking quality can be done using the plots of $\tilde{q}$. Internal stability can be assumed if the graphs of q do not blow up, t_s is finite and $\tilde{u}$ has a regular graph such that $|\tilde{u}| \leq \bar{\zeta}$. To accept that t_s is finite, the simulation stop time has to be large as compared to t_s. The measure of computational complexity is based on two parameters: t_s and κ. The larger the t_s the more constraints in (16). The larger the κ the more often the controller processor is loaded with a computationally expensive procedure.

The second focal area is the most important because the if answers to the key questions of the paper will be

[2]The source of the package Sumnis is available on request from any of the authors of this article

provided at this stage. The experimenting process goes along the same line as in the previous case except that bounded disturbances are present. Contrary to [7] the dynamics of actuators were treated marginally, because it is not possible to derive $\bar{\zeta}$ in this case. The disturbances were modelled by sinusoidal signals usually. This choice is a compromise between the experiment complexity and variety of possible signals (see e.g. [3,11–13]) covered by $\bar{\zeta}$.

The last three focal areas are of technical nature. However, they might provide a certain insight into the controller. The basis for formulation of conclusions here are the graphs of θ, $\tilde{\theta}$ and the values of the parameter κ.

4. Simulation experiments based results

Using the Sumnis software many simulation experiments[3] have been carried out, so that reliable statements to answer questions posed in Section 2 could be formulated.

4.1. Controller performance: no disturbance case. Consider the case without external disturbances and structural uncertainty, i.e., $\zeta \equiv 0$. Recall that this assumption coincides with the necessary conditions for adaptive feedback linearizing control [5]. The falsification controllers procedure carries the parameters vector θ from the initial value to the unfalsified one (i.e., representing an unfalsified controller) very fast, within a finite time interval. In other words, the number of invocations of the linear programming procedure (16), κ, is finite, and besides t_s is small. θ is a discontinuous function of time. The steady value of θ usually differs a little from θ_*. The control objective is achieved. The tracking error does not converge to zero. However, its value is very small. The falsifying signal $\tilde{u}$, after a certain time instant, remains significantly smaller in its absolute value than the disturbance prior knowledge parameter $\bar{\zeta}$.

Typical behaviour of the considered control systems is illustrated by Fig. 2, Fig. 3 and Fig. 4. The basic experimental settings have been collected in Tab. 1. It follows from Fig. 4 that parameter estimation is indeed rapid: $t_s < 1$ s and $\kappa = 5$. The parameter adjustment is very precise. The magnitude of the falsification signal $\tilde{u}$ in Fig. 3 is significantly smaller than the $\bar{\zeta}$ for $t > 1$. The graph of $\tilde{u}$ entitles us to state that the controller active at $t = 1$ will not be falsified in the future. Figure 2 shows that the signals in the system are bounded and the tracking error is very small.

Table 1
Stäubli RX-90 – simulation settings. The no disturbance case

manipulator	Stäubli RX-90
	$\theta_* = [\,16.825\ 0.375\ 1.65\ 0.2625\ 9.825\ \dots$
	$\dots\ 6.15\ 6.35\ 3.5\ 0.25\ 0.525\,]$
	$\dot{q}(0) = [\,0\ 0\ 0\,]^T, \quad q(0) = [\,0\ 0\ 0\,]^T$
controller	$\lambda = 1$
	$\theta(0) = [\,0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\,]^T$
	$q_d(t) = [\,\sin(t)\ \cos(t)\ \sin(t)\,]^T$
disturbances	$\zeta \equiv 0$, $\tau = 0$, $\bar{\zeta} = [\,0.1\ 0.1\ 0.1\,]^T$

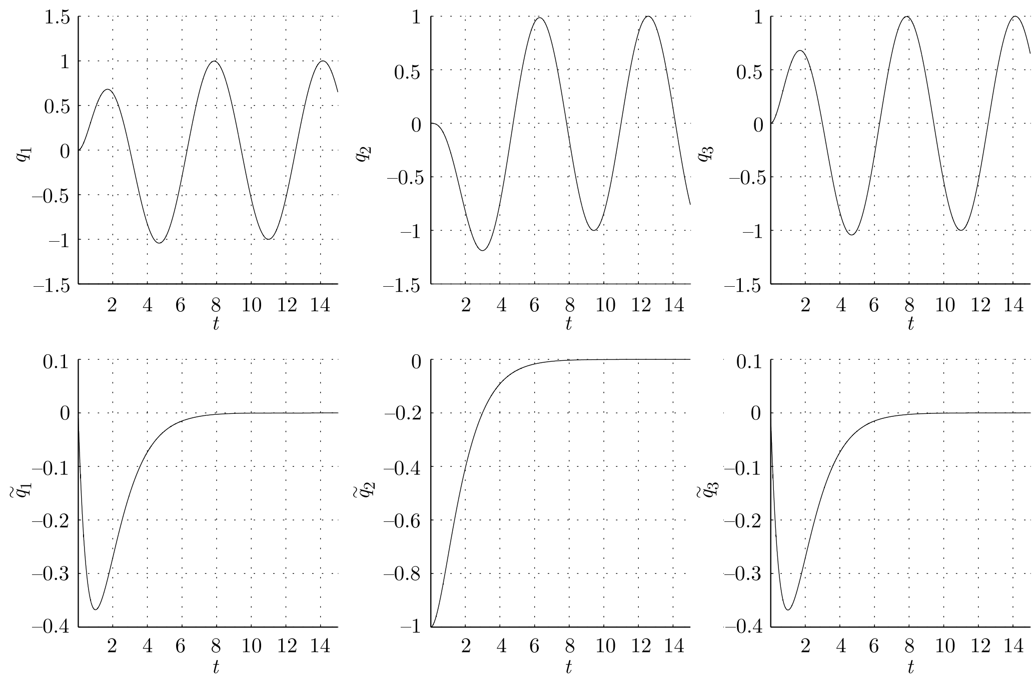


Fig. 2. Stäubli RX-90 – tracking errors. The no disturbance case

[3]The computer simulations were carried out using the software of the Wroclaw Center of Networking and Supercomputing.

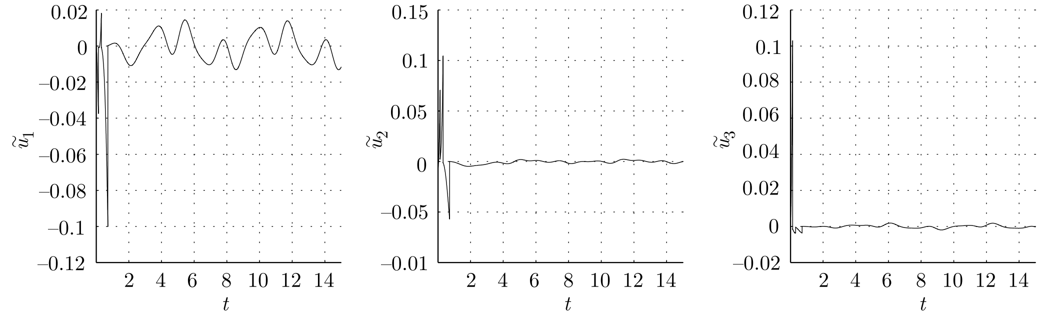


Fig. 3. Stäubli RX-90 – controller falsification. The no disturbance case

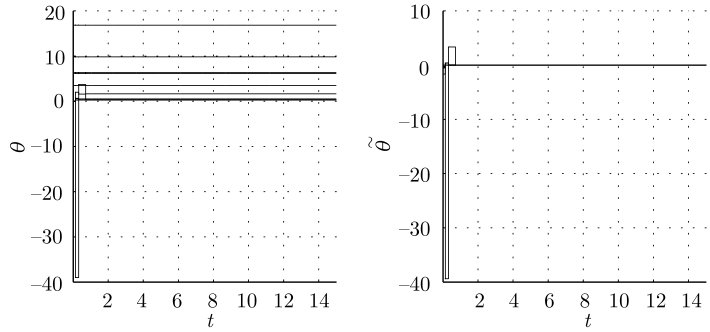


Fig. 4. Stäubli RX-90 – the estimates and the estimation errors. The no disturbance case

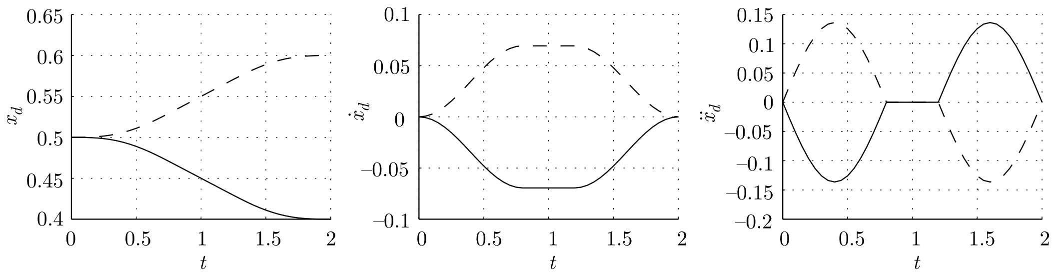


Fig. 5. EDDA the reference trajectory in the task space.The line styles: dashed and solid correspond to the X, Y coordinates respectively. The Z coordinate of the reference trajectory is equal to zero

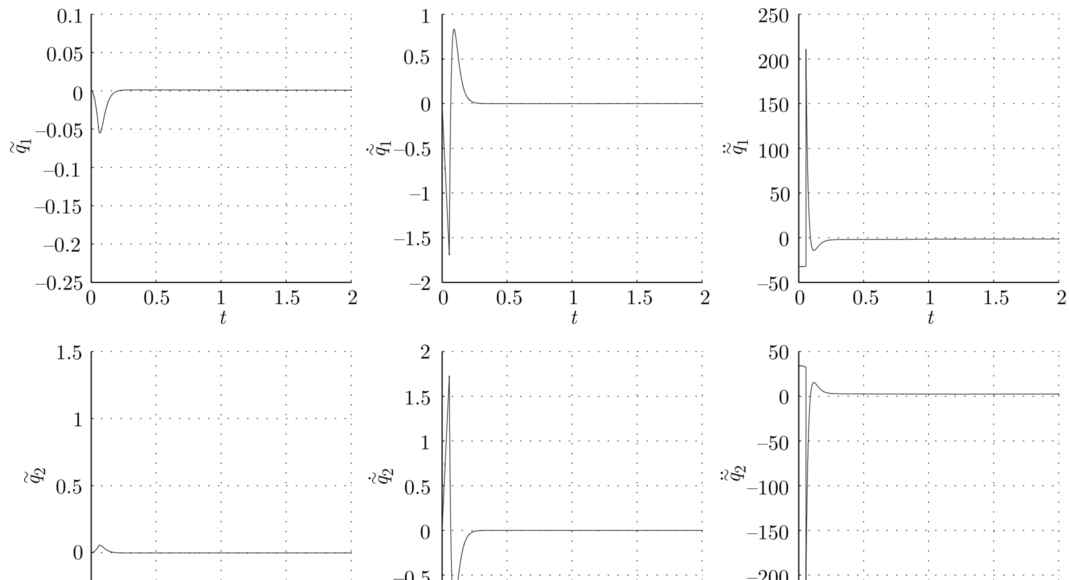


Fig. 6. EDDA – tracking errors. The no disturbance case

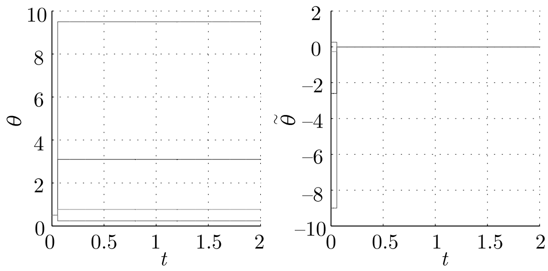


Fig. 7. EDDA – the estimates and the estimation errors. The no disturbance case

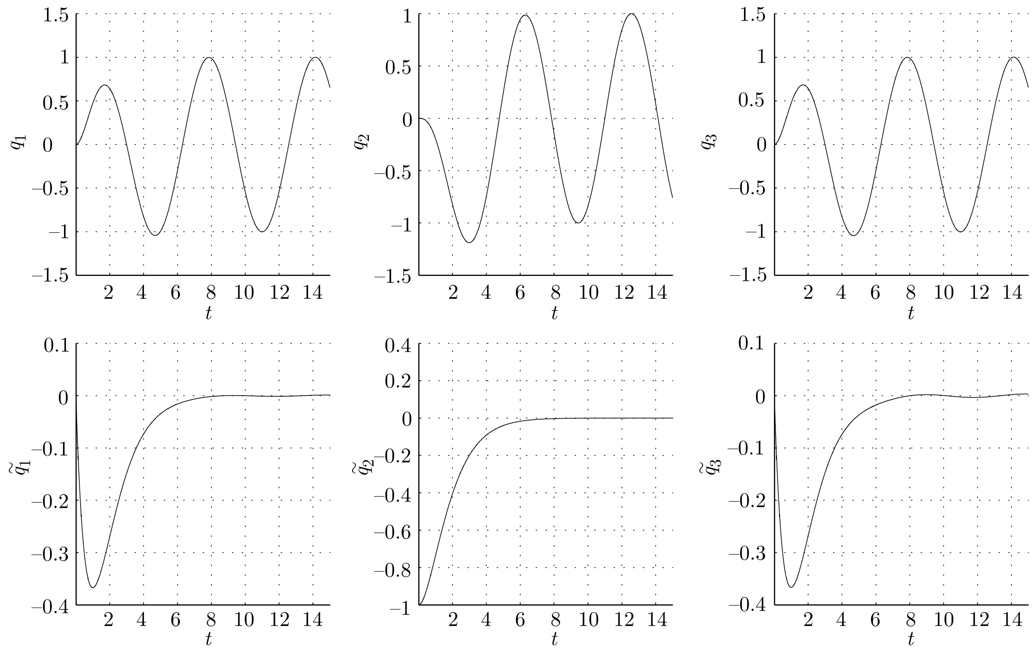


Fig. 8. Stäubli RX-90 – positions and tracking errors. The disturbance case

Table 2
EDDA - simulation settings. The no disturbance case

manipulator	EDDA $\theta_* = [\,3.1\ 9.5\ 0.24\ 0.77\,]^T$ $\dot{q}(0) = [\,0\ 0\,]^T$
controller	$\lambda = 40$ $\theta(0) = [\,0.5\ 0.5\ 0.5\ 0.5\,]^T$
disturbances	$\zeta \equiv 0$, $\tau = 0$, $\bar{\zeta} = [\,1\ 1\,]^T$

Interesting behaviour of the system controlled according to the unfalsified control strategy can be observed in Fig. 6 and Fig. 7. The simulation experiment settings have been gathered in Tab. 2. Notice, that the manipulator has the simplest structure of all the manipulators used in our experiments. The reference trajectory is defined in the task space and is shown in Fig. 5 (a straight line). It follows from Fig. 7 that $\kappa = 1$ and $t_s < 0.2$ s. The active unfalsified controller for $t > t_s$ is practically the same as $\mathsf{K}(\theta_*)$ in spite of the large values of $\bar{\zeta}$. Due to the large value of λ the tracking errors, presented in Fig. 6, converge to zero very quickly. This example is not representative in general, however, it indicates how the unfalsified controller can cope very well with manipulators with a simple structure.

4.2. Controller performance: bounded disturbance case. Consider the model of a manipulator described in Section 2, i.e., with bounded external disturbances and structural uncertainty. The falsification controllers procedure carries the parameters vector θ from the initial value to the unfalsified one (ie., representing an unfalsified controller) very quickly, in a finite time interval. The number of invocations of the linear programming procedure (16), κ, is finite and relatively small. t_s is also small. θ is a discontinuous function of time. The steady value of θ can differ from θ_* significantly. The control objective is achieved. The tracking error does not converge to zero. The falsifying signal $\tilde{u}$, after a certain time instant, remains small in its absolute value compared to the disturbance prior knowledge parameter $\bar{\zeta}$. However, this signal is larger than in the non-disturbance case.

Typical behaviour of the manipulator controlled according to the Tsao & Safonov strategy in the presence of disturbances is illustrated by Fig. 8, Fig. 9 and Fig. 10.

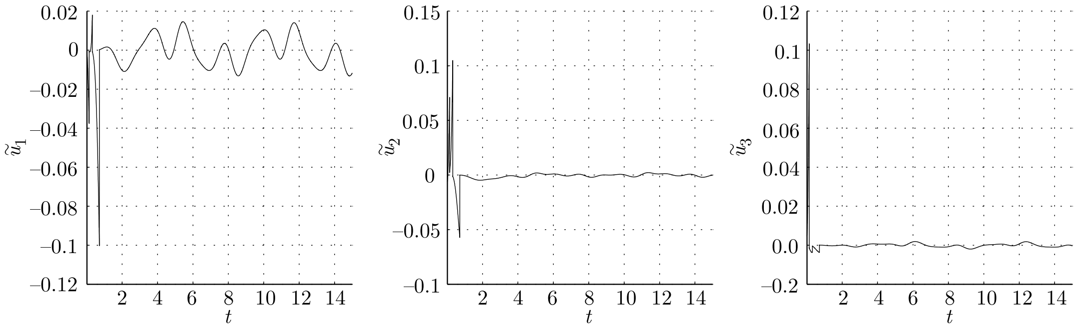


Fig. 9. Stäubli RX-90 – controller falsification signals. The disturbance case

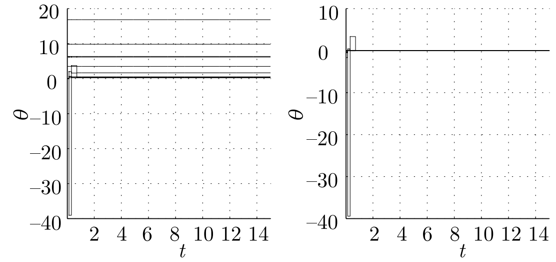


Fig. 10. Stäubli RX-90 – the estimates and the estimation errors. The disturbance case

The experimental settings are the same as in Section 4.1 (see Table 1) except that $\zeta = [0.05(\sin(0.25t) + \cos(4t))\ \ 0.04(\sin(1.25t) + \cos(0.8t))\ \ \ 0.03(\sin(t) + \cos(t))]^T$, $\bar{\zeta} = [0.1, 0.1, 0.1]^T$. It follows from Fig. 10 that as in the non-disturbance case the parameters adjustment is rapid, $\kappa = 5$, $t_s < 1$. Figure 9 shows that the falsifying signal $\tilde{u}$ is clearly inside the range $[-\bar{\zeta}, \bar{\zeta}]$ and that there are no prerequisites suggesting that in the future the active unfalsified controller at $t = 1$ will be falsified. The accuracy of the parameter adjustment is worse than in the non-disturbance case. Figure 8 shows that tracking is worse than in the non-disturbance case but it is acceptable.

Table 3
SCARA – simulation settings. The disturbance case

manipulator	SCARA $\theta_* = [\,3.26125\ 0.025\ 16\ 36\ 14.4\ 5.8\ 6.2\ 0.2\ \ldots$ $\ldots\ 3.52625\ 0.11375\ 23.04\ 2.778\ 0.005\,]^T$ $\dot{q}(0) = [\,0\ 0\ 0\ 0\,]^T, \quad q(0) = [\,0\ 0\ 0\ 0\,]^T$
controller	$\lambda = 1$ $\theta(0) = [\,0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ \ldots$ $\ldots\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\,]^T$ $q_d(t) = [\,\sin(t)\ \cos(t)\ \sin(t)\ \cos(t)\,]^T$
disturbances	$\zeta = [\,0.5(\sin(0.25t){+}\cos(4t))\ \ldots$ $\ldots\ 0.4(\sin(1.25t){+}\cos(0.8t))\ \ldots$ $\ldots\ 0.3(\sin(t){+}\cos(t))\ 0.2(\sin(0.75t){+}\cos(1.33t))\,]^T$, $\tau = 0,\ \bar{\zeta} = [\,0.5\ 0.5\ 0.5\,]^T$

Figures 11, 12 and 13 show the performance of the Tsao & Safonov controller when the manipulator structure is complex. The experimental settings are presented in Table 3.

Figure 13 shows that parameters adjustment is rapid but not so fast as in the case of less structurally complex manipulator Stäubli RX-90. Here $\kappa = 5$, $t_s < 2.5$ s. Moreover, comparison of Fig. 13 and Fig. 10 shows that the pa rameter adjustment is less precise in the case of SCARA. The graphs in Fig. 12 suggest that no controller falsification is expected for $t > 2.5$. Figure 11 shows positions q and the tracking errors $\tilde{q}$. These signals are bounded. Tracking in the first three joints is acceptable while it is

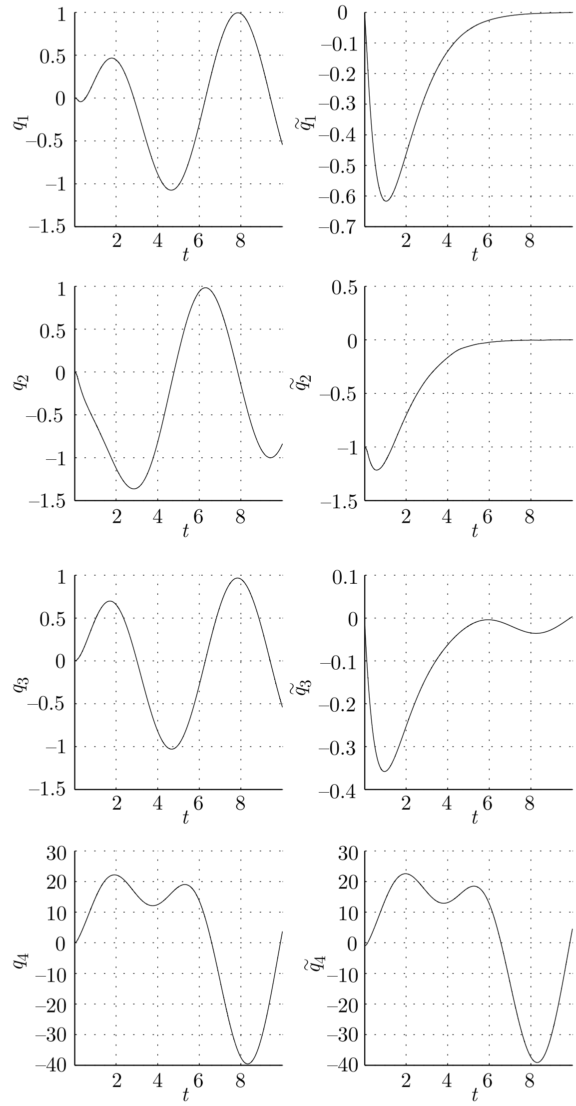


Fig. 11. SCARA – positions and tracking errors. The disturbance case

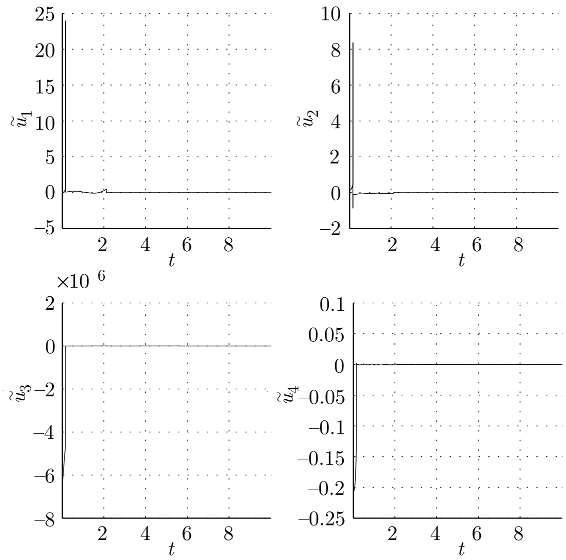


Fig. 12. SCARA – controller falsification signals. The disturbance case

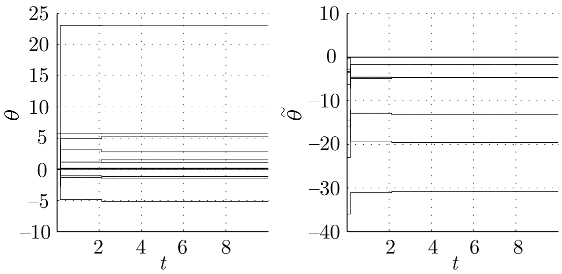


Fig. 13. SCARA – the estimates and the estimation errors. The disturbance case

not in the last joint. However, the experiments show that with this level of disturbances tracking can be improved by enlarging λ. In general the more complex manipulator structure the smaller disturbance is allowed and the higher prior knowledge about this disturbances $\bar{\zeta}$ is required to preserve the acceptable level of tracking.

4.3. Disturbances prior information and the computational complexity. In spite of the absence of external disturbances or unmodelled dynamics, the value of each component of parameter $\bar{\zeta}$ has to be positive. The effect of this parameters on the unfalsified controller behavior can be summarized as follows:

It is possible, by lowering the value of $\bar{\zeta}$,

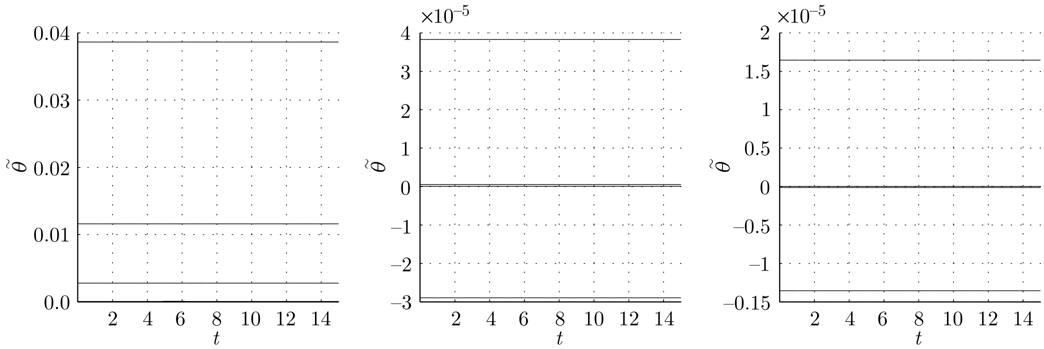
- to enlarge κ, the number of invocations of the linear programming procedure (16),
- to lengthen t_s, the time of reaching the steady value by θ
- to decrease the value of the difference between θ_* and the steady value of θ.

The more complex manipulator the larger the sensitivity to changes of $\bar{\zeta}$.

The above observations are illustrated by Figs. 14, 15, 16 and Table 4.

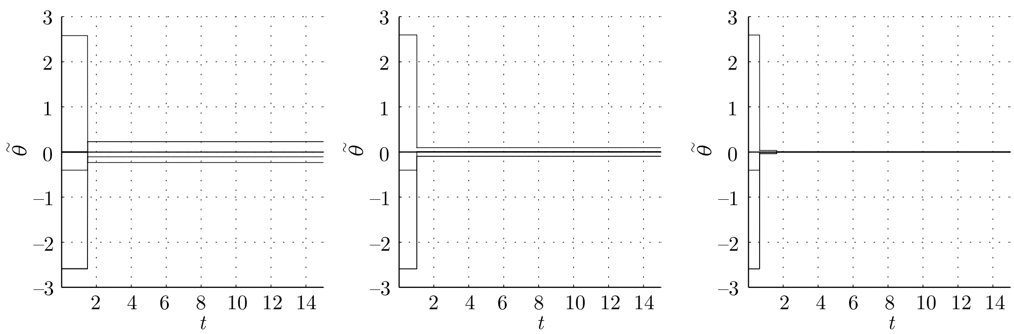
Table 4
Dependence κ on $\bar{\zeta}$

$\bar{\zeta}$	0.5	0.1	0.01	
	3	3	3	EDDA – Figure 14
κ	4	5	6	iRp-6 – Figure 15
	5	7	20	SCARA – Figure 16



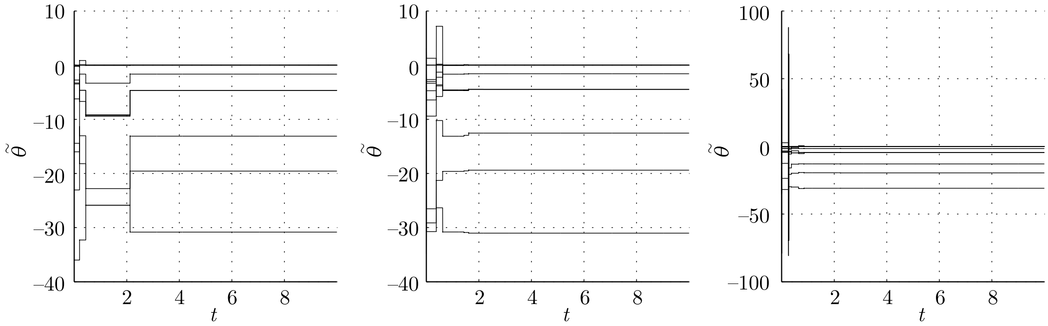
(a) $\bar{\zeta} = 0.5$ (b) $\bar{\zeta} = 0.1$ (c) $\bar{\zeta} = 0.01$

Fig. 14. EDDA – $\bar{\zeta}$ and parameters estimation errors. The no disturbance case



(a) $\bar{\zeta} = 0.5$ (b) $\bar{\zeta} = 0.1$ (c) $\bar{\zeta} = 0.01$

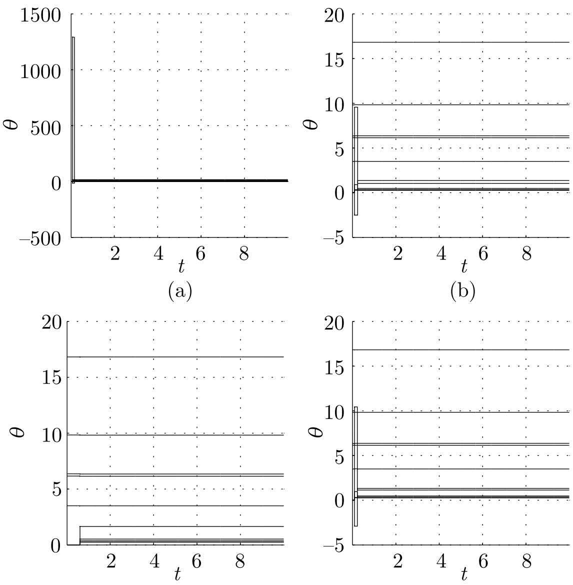
Fig. 15. IRp-6 – $\bar{\zeta}$ and parameters estimation errors. The no disturbance case



(a) $\bar{\zeta} = 0.5$ (b) $\bar{\zeta} = 0.1$ (c) $\bar{\zeta} = 0.01$

Fig. 16. SCARA – $\bar{\zeta}$ and parameters estimation errors. The no disturbance case

4.4. Closed-loop assigned spectrum and the computational complexity. Enlarging λ decreases the time interval in which θ reaches the steady value.The number of invocations of the linear programming procedure is to a small extent sensitive to the changes of the value of the parameter λ regardless of the presence of disturbances.



(a) (b) (c) (d)

Fig. 17. Stäubli RX-90 – effect of λ on θ

Table 5
Stäubli RX-90 – dependence of $\tilde{\theta}$ on λ

Figure 17, subfigure	(a)	(b)	(c)	(d)
λ	1	5	1	5
disturbances	no	no	yes	yes
κ	2	4	3	4
t_s	1	0.25	0.75	0.3

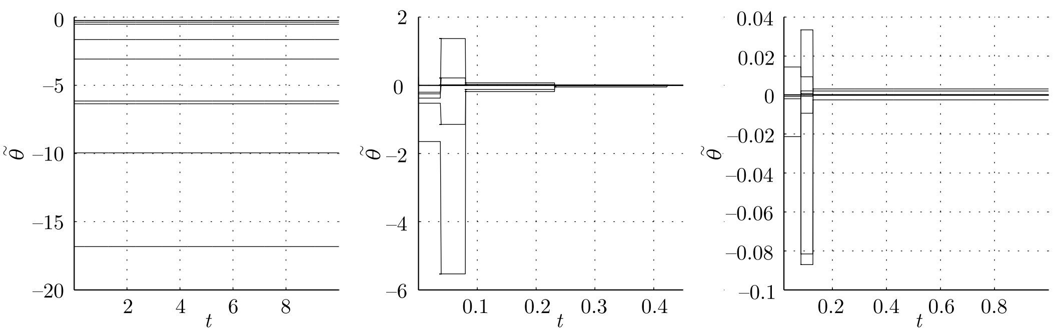
This observation is illustrated by Fig. 17 and Table 5. The non-zero disturbances are similar to those of Section (4.2). The only difference is the amplitude which is ten times larger in this case.

4.5. The reference trajectory and the accuracy of estimates. Enriching the spectrum of the reference trajectory with harmonics decreases t_s, the norm of the difference of θ_* and the steady value of θ without significant effect on the number of invocations of the linear programming algorithm (16) provided $\bar{\zeta}$ is small.

This observation is illustrated by Fig. 18 and Table 6. The basic simulation settings were following: $\lambda = 1$, $\zeta \equiv 0$, $\bar{\zeta} = [\,1\ 1\ 1\,]^T$, $\tau_s = 0$.

Table 6
Stäubli RX-90 – κ and the reference trajectory

Figure 18, subfigure	$q_d(t)$	κ
(a)	$\left[\,0\ \frac{\pi}{4}\ -\frac{\pi}{4}\,\right]^T$	1
(b)	$\left[\sum_{k=1}^{5}\sin(kt)\ \sum_{k=1}^{5}\cos(kt)\ \sum_{k=1}^{5}\sin(kt)\right]^T$	7
(c)	$\left[\sum_{k=1}^{10}\sin(kt)\ \sum_{k=1}^{10}\cos(kt)\ \sum_{k=1}^{10}\sin(kt)\right]^T$	9



(a) (b) (c)

Fig. 18. Stäubli RX-90 – effect of the reference trajectory on the parameters θ

5. Conclusions

In this article we present and analyze the results of simulation studies of robotic systems with controllers designed according to the principles established by the Tsao and Safonov in the unfalsified control strategy, [6, 7]. Compared with the similar preliminary investigations reported in [7] the range of research discussed here is significantly wider. In particular, more manipulators and with a more complex structure were used during experiments so that the formulated conclusions are more valuable for a robotican. The experiments were carried out systematically with the help of the Sumnis software, especially built for the purposes of the experiments.

It has been concluded, as a result of numerous experiments, that in the approximately real conditions the controllers falsification algorithm finds an unfalsified controller in a finite number of steps, usually small, in a very short time. Particularly, the phenomena of chattering has never been observed. In consequence the control system is internally stable and the computational complexity of the controller is relatively low. Moreover, several conclusions referring to the effect of various parameters on the behavior of the controller were formulated. They enable the reader to acquire certain intuition about the behavior of an unfalsified controller.

The obtained results indicate expedience of widest interest in the Tsao & Safonov control strategy, in particular in the context of practical implementation and theoretical explanation of all aspects related to computational complexity and stability. The extra prerequisite motivating deeper interest in this controller is a very interesting design strategy, significantly different from the strategies commonly used in robotics.

Experimental study of the unfalsified control of manipulators is under way. The results concerning practical aspects of this strategy will be available in the foreseeable future.

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